

# **Chance Constrained Land Allocation Planning Problems Of Agricultural Systems In Inexact Decision Making Environment**

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## **Abstract**

This paper describes how the fuzzy goal programming (FGP) can be efficiently used for modelling and solving land allocation problems having chance constraints for optimal production of seasonal crops of agricultural system in inexact environment. The environment in which we live and work is highly imprecise in nature. Unpredictability of the rainfall conditions and unavailability of fresh irrigation water supply due to socio-economic conditions is a matter of concern in the complex real world agricultural situations.

In the proposed model, utilization of total cultivable land, different farming resources, achievement of the aspiration levels of production of seasonal crops are fuzzily described. Water supply as a productive resource and the socio-economic constraints are described probabilistically in the decision making environment. The land-use planning problem for production of the five principal crops such as Paddy, Wheat, Mustard, Potato, Pulses in three different seasons such as the crop-cycles Pre-kharif, Kharif and Rabi successively throughout the planning year of the District Bardhaman of West Bengal (W.B.) in India is considered to illustrate the proposed FGP model.

In the solution process, achievement of the highest membership value (unity) of the membership goals defined for the fuzzy goals of the problem to the extent possible on the basis of the needs and desires of the decision maker (DM) is taken into account in the decision making horizon. The potential use of the approach is demonstrated by a case example of the Bardhaman district, West Bengal (W. B.), INDIA.

**Keywords:** Agricultural Planning, Chance Constrained Programming, Fuzzy Programming, Fuzzy Goal Programming, Goal Programming.

## **1. Introduction**

The history of agriculture shows that the significant improvements in the agricultural techniques and technology were taken place from the 12th to the 13th century. Further, crop yields peaked in the 13th century, and stayed more or less steady until the 18<sup>th</sup> century. Now, with the growth of population and Cultural Revolution, although cropping and water supply system were improved a lot prior to middle of the last century, the Green Revolution has taken place during 1960s. Planning models were then developed for water supply and farming systems to meet the needs in society. To meet the human needs, water is tapped at the point between precipitations on to land and discharge into the ocean. Only 2.5% of the water of our planet is suitable for drinking and irrigation, out of which 98.6 % water is tapped in glacier, snow, ground water and soil moisture. India gets 0.7% of world water precipitation (4000 Km<sup>3</sup>). In India, rainfall is spatially and temporally skewed and uneven which sets serious challenges for water resource management. The state West Bengal is endowed with an excellent geophysical location - starting from the great Himalayan Range in the north to the Bay of Bengal in south. West Bengal is bestowed with 7.5% of water resources of India and renders home for 8% of National Populations. Due to Highest population density (904/Sq Km, 2011), development needs and extensive irrigation for agriculture based economy has created water stress in West Bengal. The annual per capita availability of fresh water in 1961 was 5,177 cubic meters, which declined to 1,869 cubic meters in 2001. It is likely to fall further to 1,341 cubic meters in 2025. The present water crises in West Bengal are due to misuse and abuse of water. Now, in the agricultural production planning context, it is worthy to mention that water is the main resource in crop production system,

and adequate and sustainable supply of water depend solely on the amount of rainfall in all the seasons throughout a year. As such, rain water supply to meet various needs is considered to be probabilistically uncertain in nature. Further, irrigation water supply is a complicated issue which involves socio-economic and environmental impacts. Various uncertainties are associated with demand patterns and availability of water. Again, it is worthy to mention that industrial emissions have now become a great threat to the environment of the planet Earth. It has now become a great challenge to obtain fresh water for the farmers. Particularly, agricultural sectors are facing water scarcity challenges along with serious threat from water pollution and climate change issues. It is also worthy to mention that the balanced supply of different farming resources such as Fertilizers is very important in farm planning problems. Fertilizers are organic or inorganic substances, either natural or synthetic, used to supply macro nutrients (such as nitrogen (N), phosphorus (P), potassium (K), Calcium (Ca), Sulphur (S), Magnesium (Mg) etc.) and micro nutrients (such as, Boron (B), Manganese (Mn), Zinc (Zn) etc.) that are essential for plant growth. They are the most effective means of increasing crop production and of improving the quality of food and fodder. Over-application of fertilizer is a common problem in India particularly West Bengal. Sustainable crop production system as a part of sustainable agriculture, seeks to optimize the management and use of internal production inputs (i.e., on-farm resource) and to minimize the use of production inputs (i.e., off farm resources), such as purchased fertilizers and pesticides, wherever feasible and practicable, to lower production cost, to avoid pollution of surface and groundwater, to reduce pesticide residues in food, to reduce a farmer's overall risk and to increase both short-and long-term farm profitability. Intensive crop cultivation with modern high-yielding varieties using high analysis fertilizers has induced secondary and micronutrient deficiencies in soils. Many times, these hidden nutrient deficiencies are not identified and corrected. These hidden nutrient deficiencies, if not identified and removed beforehand, will limit crop yield. In an agricultural planning situation, since optimal production of crops highly depends on the proper allocation of land in different seasons for cultivating the crops and adequate supply of productive resources and optimal water management, most of the farms planning problems are multiobjective in nature. Now, in case of the proposed problem, since various objectives associated with cropping plan are imprecise in nature and they are generally non- commensurable, simultaneous optimization of them is not always possible in the decision situation. As such, it seems that FGP as a goal satisficer, rather than objective optimizer can be efficiently used to farm management problems. So, a great challenge is faced by the decision makers to make a proper plan of water resources and balance the fertilizer inputs for the survival and sustainable growth of the agricultural systems.

The mathematical programming models to agricultural production planning have been widely used since Heady [1] in 1954 demonstrated the use of linear programming (LP) for land allocation to cropping plan in agricultural system. From the mid-'60s to '80s of the last century, different Linear Programming (LP) models studied [2, 3] for farm planning has been surveyed by Glen in [4] in 1987. Since most of the farm planning problems are multiobjective in nature, the goal programming (GP) methodology in [5] as a prominent tool for multiobjective decision analysis has been efficiently used to land-use planning problems [6] in the past.

Although, GP has been widely accepted as a promising tool for multiobjective decision making (MODM), the main weakness of the conventional GP methodology is that the aspiration levels of the goals need be stated precisely. To overcome the above difficulty of imprecise in nature of them, fuzzy programming (FP) approach in [7] to farm planning problems has been deeply studied [8] in the past. The FGP approach [9] as an extension of conventional GP to agricultural production planning problems has also been studied by Pal et al. [6,10] in the past.

Now, in most of the real-world decision situations, the DMs are often faced with the problem of inexact data due to inherent uncertain in nature of the resource parameters involved with the problems. To deal with the probabilistically uncertain data, the field of stochastic programming (SP) has been studied [11] extensively and applied to various real-life problems [12, 13] in the past.

The use of chance constrained programming (CCP) to fuzzy MODM problems has also been studied by Pal et al. [14] in the recent past. However, consideration of both the aspects of FP and SP for modeling and solving real-life decision problems has been realized in the recent years from the view point of occurrence of both the fuzzy and probabilistic data in the decision making environment. Although, fuzzy stochastic programming (FSP) approaches to chance constrained MODM problems have been investigated [15] by active researchers in the field, the extensive study in this area is at an early stage.

Now, in the agricultural production planning context, it is worthy to mention that the sustainable supply of water depends solely on the amount of rainfall in all the seasons throughout a year. As such, water supply to meet various needs is very much stochastic in nature.

Although, several modeling aspects of water supply system have been investigated [12] in the past, consideration of probabilistic parameters to the agricultural systems in fuzzy decision environment is yet to be circulated in the literature.

In this article, utilization of total cultivable land, different farming resources, achievement of the aspiration levels of production of seasonal crops are fuzzily described. The water supply as a productive resource and certain socio-economic constraints are described probabilistically in the decision making environment.

In the solution process, achievement of the highest membership value (unity) of the membership goals defined for the fuzzy goals of the problem to the extent possible on the basis of the needs and desires of the decision maker (DM) is taken into account in the decision making horizon.

The potential use of the approach is demonstrated by a case example of the Bardhaman District, West Bengal (W. B.), INDIA.

Now, the general chance constrained FGP formulation is presented in the Section

**2 Problem Formulation**

The general form of a fuzzy MODM problem with chance constrained can be stated as:

Find  $X (x_1, x_2, \dots, x_n)$  so as to

$$\text{satisfy } Z_k(X) \begin{pmatrix} \gtrsim \\ \lesssim \end{pmatrix} g_k, \quad k = 1, 2, \dots, K. \tag{1}$$

(1)

$$\text{subject to } X \in S \{X \in R^n \mid \text{Pr}[H(X) \geq b] \geq p, X \geq 0, H, b \in R^m\}, \tag{2}$$

where  $X$  is the vector of decision variables in the bounded feasible region  $S (\neq \Phi)$ , and where  $\gtrsim$  and  $\lesssim$  indicate the fuzziness of  $\geq$  and  $\leq$  restrictions, respectively, in the sense of Zimmermann [7], and where  $g_k$  be the imprecise aspiration level of the  $k$ -th objective. Pr stands for probabilistically defined (linear / nonlinear ) constraints set  $H(X)$ ,  $b$  is a resource vector, and  $p$  ( $0 < p < 1$ ) is the vector of satisficing probability levels of the defined constraints.

Now, to formulate the FGP model of the problem, the fuzzy goals in (1) are first characterized by their membership functions in [7] to measure the degree of achievement of the goals. Then, they are transformed into membership goals [16] by assigning the highest membership value (unity) as the aspiration level for goal achievement and introducing under- and over- deviational variables to each of them. Again, the probabilistic constraints are converted into their deterministic equivalent to employ the proposed approach in the process of solving the problem.

**2.1 Construction of Membership Goals of Fuzzy Goals**

The membership goal expression of the membership function  $\mu_k(X)$  defined for the fuzzy goal  $Z_k(X)$  &  $g_k$  appears as [16]:

$$\mu_k(X) : \frac{Z_k(X) - g_{lk}}{g_k - g_{lk}} + d_k^- - d_k^+ = 1, k \in K_1 \tag{3}$$

where,  $g_{lk}$  and  $(g_k - g_{lk})$  represent the lower tolerance limit and tolerance range, respectively, for achievement of the associated k-th fuzzy goal. Also,  $d_k^- \geq 0$  and  $d_k^+ \geq 0$  are the under- and over-deviational variables, respectively, of the k-th membership goal  $\mu_k(X)$ .

Similarly, the membership goal expression for the fuzzy goal  $Z_k(X)$  .  $g_k$  takes the form:

$$\mu_k(X) : \frac{g_{uk} - Z_k(X)}{g_{uk} - g_k} + d_k^- - d_k^+ = 1, k \in K_2 \tag{4}$$

where,  $g_{uk}$  and  $(g_{uk} - g_k)$  represent the upper tolerance limit and tolerance range, respectively, for achievement of the associated k-th fuzzy goal, and where  $k_1 \cup k_2 = \{1, 2, \dots, K\}$  with  $k_1 \cap k_2 = \Phi$ .

**2.2 Deterministic Equivalent of Chance Constraints**

The deterministic equivalent of a chance constraint depends on randomness and probability distribution of the parameters involved with the constraints. In the present decision situation, the resource vector  $b$  in (2) is taken as normally distributed random parameter.

Then, the deterministic equivalent of the i-th constraint takes the form [15]:

$$H_i(X) \geq E(b_i) + F_i^{-1}(p_i) \sqrt{\text{var}(b_i)}, i=1, 2, \dots, m \tag{5}$$

where,  $E(b_i)$  and  $\text{var}(b_i)$  represent mean and variance of  $b_i$  and  $F^{-1}(\cdot)$  represents the inverse of the probability distribution function  $F(\cdot)$  of standard normal variate.

Now, the general FGP model of the problem is presented in the following Section 2.3.

**2.3 FGP Model Formulation**

In a fuzzy decision making situation, the aim of the decision maker ( DM) is to achieve the highest membership value of each of the defined membership goals to the extent possible by minimizing the under deviational variables of each of them, and that depends on the needs and desires of the DM.

In a fuzzy decision making situation, the aim of the DM is to achieve the highest membership value of each of the defined membership goals to the extent possible by minimizing the under deviational variables of each of them, and that depends on the needs and desires of the DM.

The FGP model formulation under a pre-emptive priority structure can be presented as [10]:

Find  $X(x_1, x_2, \dots, x_n)$  so as to

Minimize  $Z = [P_1(d^-), P_2(d^-), \dots, P_r(d^-), \dots, P_R(d^-)]$

and satisfy the membership goals in (3) and (4), subject to the system constraints set in (5); where  $Z$  represents the vector of  $R$  priority achievement function.  $P_r(d^-)$  is a linear function of the weighted under-deviational variables, where  $P_r(d^-)$  is of the form

$$P_r(d^-) = \sum_{k=1}^K w_{rk}^- d_{rk}^-, \quad k = 1, 2, \dots, K ; (R \leq K),$$

where  $d_{rk}^-$  is renamed for  $d_k^-$  to represent it at the r-th priority level,  $w_{rk}^- (>0)$  is the numerical weight associated with  $d_{rk}^-$  and it designates the weight of importance of achieving the aspired level of the k-

th goal relative to other which are grouped at the r-th priority level and where  $w_{rk}$  values are determined as [16] :

$$w_{rk}^- = \begin{cases} \frac{1}{(g_k - g_{kl})_r}, & \text{for the defined } \mu_k(\mathbf{X}) \text{ in (3)} \\ \frac{1}{(g_{ku} - g_k)_r}, & \text{for the defined } \mu_k(\mathbf{X}) \text{ in (4)} \end{cases}$$

where  $(g_k - g_{kl})_r$  and  $(g_{ku} - g_k)_r$  are used to present  $g_k - g_{kl}$  and  $g_{ku} - g_k$  respectively, at the r-th priority level.

Now, the FGP model formulation of the proposed problem is presented in the Section 3.

### 3 FGP Model Formulation of the Problem

The decision variables and different types of parameters involved with the problem are defined first in the following Section 3.1.

#### 3.1 Definition of Decision Variables and Parameters

##### (a) Decision Variable:

$l_{cs}$  = Allocation of land for cultivating the crop c during the season s,  $c = 1, 2, \dots, C$ ;  $s = 1, 2, \dots, S$ .

##### (b) Productive resource parameters:

- Fuzzy resources:

$LA_s$  = Total farming land (hectares (ha)) currently in use for cultivating the crops in the season s.

$MH_s$  = Estimated total machine hours (in hrs.) required during the season s.

$MD_s$  = Estimated total man-days (in days) required during the season s.

$F_f$  = Estimated total amount of the fertilizer f ( $f = 1, 2, \dots, F$ ) (in quintals (qtls.)) required during the planning year.

$RS$  = Estimated total amount of cash (in Rupees (in Rupees)) required per annum for supply of the productive resources.

- Probabilistic resource:

$WS_s$  = Total supply of water (in inch / ha) required during the season s.

##### (c) Fuzzy aspiration levels:

$P_c$  = Annual production level (in qtls.) of the crop c.

$MP$  = Estimated total market value (in Rupees) of all the crops yield during the planning year.

##### (d) Probabilistic aspiration levels:

$R_{ij}$  = Ratio of annual production of the i-th and j-th crop ( $i, j = 1, 2, \dots, C$ ;  $i \neq j$ ).

$r_{ij}$  = Ratio of annual profits obtained from the i-th and the j-th crops ( $i, j = 1, 2, \dots, C$ ;  $i \neq j$ ).

##### (e) Crisp coefficients:

$MH_{cs}$  = Average machine hours (in hrs.) required for tillage per ha of land for cultivating the crop c during the season s.

$MD_{cs}$  = Man days (in days) required per ha of land for cultivating the crop c during the season s.

$F_{fcs}$  = Amount of the fertilizer f required per ha of land for cultivating the crop c during the season

s.

$P_{cs}$  = Estimated production of the crop c per ha of land cultivated during the season s.

$A_{cs}$  = Average cost for purchasing seeds and different farm assisting materials per ha of land cultivated for the crop c during the season s.

$MP_{cs}$  = Market price (Rupees / qt1.) at the time of harvest of the crop c cultivated during the season s.

**(f) Random coefficients:**

$W_{cs}$  = Estimated amount of water consumption (in inch) per ha of land for cultivating the crop c during the season s.

**3.2 Description of Fuzzy Goals and Chance Constraints**

**(a) Land utilization goal:**

The land utilization goal for cultivating the seasonal crops appears as:

$$\sum_{c=1}^C l_{cs} \lesssim LA_s, \quad s = 1, 2, \dots, S.$$

**(b) Productive resource goals:**

- Machine-hour goal: An estimated number of machine hours is to be provided for cultivating the land in different seasons of the plan period.

The fuzzy goals take the form:

$$\sum_{c=1}^C MH_{cs} . l_{cs} \gtrsim MH_s, \quad s = 1, 2, \dots, S.$$

- Man-power requirement goals: A number of labors are to be employed through out the planning period to avoid the trouble with hiring of extra labors at the peak times.

The fuzzy goals take the form:

$$\sum_{c=1}^C MD_{cs} . l_{cs} \gtrsim MD_s, \quad s = 1, 2, \dots, S.$$

- Fertilizer requirement goals: To maintain the fertility of the soil, different types of fertilizer are to be used in different seasons in the plan period.

The fuzzy goals take the form:

$$\sum_{c=1}^C F_{fcs} . l_{cs} \gtrsim F_f, \quad f = 1, 2, \dots, F; \quad s = 1, 2, \dots, S.$$

**(c) Cash expenditure goal:**

An estimated amount of money (in Rs.) is involved for the purpose of purchasing the seeds, fertilizers and other productive resources.

The fuzzy goals take the form:

$$\sum_{s=1}^S \sum_{c=1}^C A_{cs} . l_{cs} \gtrsim RS$$

**(d) Production achievement goals:**

To meet the demand of agricultural products in society, a minimum achievement level of production of each type of the crops is needed.

The fuzzy goals appear as:

$$\sum_{s=1}^S P_{cs} \cdot J_{cs} \gtrsim P_c, \quad c = 1, 2, \dots, C.$$

**(e) Profit goal:**

A certain level of profit from the farm is highly expected by the farm decision maker.

The fuzzy profit goal appears as:  $\sum_{s=1}^S \sum_{c=1}^C (MP_{cs} \cdot P_{cs} - A_{cs}) \cdot J_{cs} \gtrsim MP$

**3.3 Description of Chance Constraints**

The different chance constraints of the problem are presented in the following Sections.

**(a) Water-supply constraints:**

An estimated amount of water need be supplied to the soil for sustainable growth of the crop *c* cultivated during the season *s*. But, water-supply resources solely depends on rainfall and so probabilistic in nature.

The water-supply constraints appear as:

$$\Pr\left[\sum_{c=1}^C W_{cs} \cdot J_{cs} \geq W_s\right] \geq p_s, \quad s = 1, 2, \dots, S.$$

where  $p_s$  ( $0 < p_s < 1$ ) denotes the satisficing level of probability for the supply of water.

**(b) Production-ratio constraints:**

To meet the demand of the primary food products in society, allocation of land for the crops production in different seasons should be made in such a way that certain ratios of total production of major crops can be maintained.

The production-ratio constraints appear as:

$$\Pr\left[\frac{\sum_{s=1}^S P_{is} \cdot J_{is}}{\sum_{s=1}^S P_{js} \cdot J_{js}} \geq R_{ij}\right] \geq p_{ij}, \quad i, j = 1, 2, \dots, C, \text{ and } i \neq j.$$

where  $p_{ij}$  ( $0 < p_{ij} < 1$ ) denotes the satisficing level of probability for the ratios of *i*-th and *j*-th crops.

**(c) Profit-ratio constraints:**

Here, similar to the case in production-ratio constraints, the profit-ratio constraints are random in nature. The profit-ratio constraints take the form:

$$\Pr\left[\frac{\sum_{s=1}^S (MP_{is} \cdot P_{is} - A_{cs}) \cdot J_{is}}{\sum_{s=1}^S (MP_{js} \cdot P_{js} - A_{cs}) \cdot J_{js}} \geq r_{ij}\right] \geq q_{ij}, \quad i, j = 1, 2, \dots, C, \text{ and } i \neq j.$$

where,  $q_{ij}$  ( $0 < q_{ij} < 1$ ) denotes the satisficing level of probability for the *i*-th and *j*-th profit-ratio.

**4 An Illustrative Example: A Case Study**

The land-use planning problem for production of the principal crops of the District Bardhaman of West Bengal (W.B.) in India is considered to illustrate the proposed FGP model. Now, the three

seasonal crop- cycles: Pre-kharif, Kharif and Rabi successively appear in W.B. during a planning year, and they designate the time periods for crop production during summer, rainy and winter seasons, respectively. The data were collected from different sources recorded District Statistical Hand Book, 2015 [17]; Economic Review; Basak, 2017 [18].

The decision variables and different types of data involved with the problem are summarized in the following Tables 1–4.

**Table 1.** Summary of the seasonal crops and decision variables

Season (s)	Pre-kharif (1)			Kharif(2)	Rabi (3)				
Crop (c)	Jute (1)	Sugarcane (2)	Aus-paddy (3)	Aman-paddy (4)	Boro-paddy (5)	Wheat (6)	Mustard (7)	Potato (8)	Pulses (9)
Variable ( $l_{cs}$ )	$l_{11}$	$l_{21}$	$l_{31}$	$l_{42}$	$l_{53}$	$l_{63}$	$l_{73}$	$l_{83}$	$l_{93}$

**Table 2:** Data description of the aspired goal levels and tolerance limits

Goal	Aspiration Level	Tolerance Limit	
		Lower	Upper
1. Land utilization ('000 hectares) :			
(i) Pre-kharif season	272.14	----	309.33
(ii) Kharif season	272.14	----	309.33
(iii) Rabi season	272.14	----	309.33
2. a) Machine-hours (in hrs.) :			
(i) Pre-kharif season	14522.30	13070.07	----
(ii) Kharif season	7253.57	6528.22	----
(iii) Rabi season	35373.01	31835.71	----
b) Man-days (days) :			
(i) Pre-kharif season	14274.1	12846.69	----
(ii) Kharif season	6513.1	5861.79	----
(iii) Rabi season	11312.2	10180.98	----
c) Fertilizer requirement (metric ton) :			
(i) Nitrogen	51.5	46.36	----



Goal	Aspiration Level	Tolerance Limit	
		Lower	Upper
(ii) Phosphate	19.7	17.73	----
(iii) Potash	19.7	17.73	----
3. Production (‘000 metric ton) :			
(a) Jute	339.660	325.152	----
(b) Sugarcane	78.798	70.919	----
(c) Rice	829.00	800	----
(d) Wheat	126.28	111.78	----
(e) Mustard	94.71	86.10	----
(f) Potato	526.18	478.34	----
(g) Rabi pulse	38.6	28.3	----
4. Cash expenditure (Rupees Lac.)	82636.4175	----	90900.0592
5. Profit (Rupees Lac.)	81470.8825	73323.794	----

**Table 3.** Data description of productive resource utilization, cash expenditure and market price.

Crops	MH <sub>s</sub>	MD <sub>s</sub>	F <sub>f</sub>			PA	CE	MP
			N	P	K			
Jute	66.72	90	40	20	20	2603	17297.00	1050
Sugarcane	166.76	123	200	100	100	70364	30887.50	1200
Aus	138.99	60	40	20	20	2256.039	14331.80	1350
Aman	66.72	60	40	20	20	2153.989	12849.20	1600
Boro	266.87	60	100	50	50	3482.690	23721.60	1200
Wheat	66.72	39	100	50	50	2187.633	11119.50	1100
Mustard	33.36	30	80	40	40	869.182	8401.40	1700
Potato	111.19	70	150	75	75	21087.719	37312.10	500
Pulses	49.06	15	20	50	20	725.641	4942.00	2200

Note: MH<sub>s</sub> = machine hours (in hrs/ha), MD<sub>s</sub> = man-days (days/ha), F<sub>f</sub> = fertilizer (kg/ha): N=Nitrogen, P = Phosphate, K = Potash; PA = production achievement (kg/ha), CE = cash expenditure (Rs / ha), MP = market price (Rs / qt).

**Table 4.** Data description of water-supply, water-utilization, production-ratio and profit-ratio.

WU (i)	Year			
	2003-2004	2004-2005	2005-2006	2006-2007
↓ 1	20	20	20	20

2	60	60	60	60
3	34	34	34	34
4	50	50	50	50
5	70	70	70	70
6	15	15	15	15
7	10	10	10	10
8	18	18	18	18
9	10	10	10	10
WS(PKS,KS,RS)	(116.93, 159.85, 264.62)	(119.42, 147.76, 335.92)	(100.44, 147.77, 243.49)	(100.96, 133.19, 224.10)
PDR(Rice and Wheat)	6.22	7.39	6	6.6
PR(Jute and Aus-paddy)	1.17	2.27	5.5	2

Note: WU(i)= Water-utilization (inch/ha) for the i-th crop (i=1,2,...,9), WS(.)= Water-supply (inch), PKS= Pre-kharif season, KS= Kharif season, RS= Rabi season, PDR= Production-ratio, PR= Profit-ratio. Now, using the data Tables 1-3, the membership functions of the defined fuzzy goals can be constructed by using the expressions in (3) and (4).

The fuzzy goals appear as follows:

**Land utilization goals:**

The membership goals for land utilization in the three consecutive seasons appear as

$$\begin{aligned} \mu_1 &: 8.32 - 0.027 (l_{11} + l_{21} + l_{31}) + d_1^- - d_1^+ = 1 && \text{(Pre-kharif)} \\ \mu_2 &: 8.32 - 0.027 (l_{21} + l_{42}) + d_2^- - d_2^+ = 1 && \text{(Kharif)} \\ \mu_3 &: 8.32 - 0.027 (l_{21} + l_{53} + l_{63} + l_{73} + l_{83} + l_{93}) + d_3^- - d_3^+ = 1 && \text{(Rabi)} \end{aligned} \tag{6} \quad \text{Productive resource goals:}$$

**Machine-hour goal**

$$\begin{aligned} \mu_4 &: 0.0459 l_{11} + 0.0383 l_{21} + 0.0957 l_{31} - 9 + d_4^- - d_4^+ = 1 && \text{(Prekharif)} \\ \mu_5 &: 0.0919 l_{21} + 0.0766 l_{42} - 8.999 + d_5^- - d_5^+ = 1 && \text{(Kharif)} \\ \mu_6 &: 0.0157 l_{21} + 0.0754 l_{53} + 0.0189 l_{63} + 0.0094 l_{73} + 0.0314 l_{83} && \text{(Rabi)} \\ &+ 0.0139 l_{93} - 9.012 + d_6^- - d_6^+ = 1 && \text{(7) Man-power goals:} \\ \mu_7 &: 0.0630 l_{11} + 0.0287 l_{21} + 0.0420 l_{31} - 9.03 + d_7^- - d_7^+ = 1 && \text{(Prekharif)} \\ \mu_8 &: 0.0629 l_{21} + 0.0921 l_{42} - 8.98 + d_8^- - d_8^+ = 1 && \text{(Kharif)} \\ \mu_9 &: 0.0362 l_{21} + 0.0530 l_{53} + 0.0345 l_{63} + 0.0265 l_{73} + 0.0619 l_{83} + 0.0133 l_{93} - 9 + d_9^- - d_9^+ = 1 && \text{(Rabi)} \end{aligned} \tag{8}$$

**Fertilizer requirement goals:**

$$\begin{aligned} \mu_{10} : & 0.0097 l_{11} + 0.0388 l_{21} + 0.0116 l_{31} + 0.0116 l_{42} + 0.0252 l_{53} + 0.0233 l_{63} + 0.0194 l_{73} \\ & + 0.0388 l_{83} + 0.0039 l_{93} - 9.001 + d_{10}^- - d_{10}^+ = 1 \quad (N) \\ \mu_{11} : & 0.0127 l_{11} + 0.0508 l_{21} + 0.0152 l_{31} + 0.0152 l_{42} + 0.0329 l_{53} + 0.00202 l_{63} \\ & + 0.0254 l_{73} + 0.0761 l_{83} + 0.0203 l_{93} - 8.89 + d_{11}^- - d_{11}^+ = 1 \quad (P) \\ \mu_{12} : & 0.0253 l_{11} + 0.0501 l_{21} + 0.0152 l_{31} + 0.0152 l_{42} + 0.0329 l_{53} + 0.0304 l_{63} \\ & + 0.0254 l_{73} + 0.0761 l_{83} + 0.0203 l_{93} - 9 + d_{12}^- - d_{12}^+ = 1 \quad (K) \end{aligned} \quad (9)$$

**Cash expenditure goal:**

$$\begin{aligned} \mu_{13} : & 11 - (0.0209 l_{11} + 0.0374 l_{21} + 0.0173 l_{31} + 0.0016 l_{42} + 0.0287 l_{53} \\ & + 0.0134 l_{63} + 0.0102 l_{73} + 0.0452 l_{83} + 0.0059 l_{93}) + d_{13}^- - d_{13}^+ = 1 \end{aligned} \quad (10)$$

**Production achievement goals:**

$$\begin{aligned} \mu_{14} : & 0.0778 l_{31} + 0.0742 l_{42} + 0.01201 l_{53} - 27.5862 + d_{14}^- - d_{14}^+ = 1 \quad (\text{Rice}) \\ \mu_{15} : & 0.200 l_{11} - 22.4119 + d_{15}^- - d_{15}^+ = 1 \quad (\text{Jute}) \\ \mu_{16} : & 0.1509 l_{63} - 7.7089 + d_{16}^- - d_{16}^+ = 1 \quad (\text{Wheat}) \\ \mu_{17} : & 3.93 l_{21} - 5.9 + d_{17}^- - d_{17}^+ = 1 \quad (\text{Sugercan}) \\ \mu_{18} : & 0.1009 l_{73} - 10 + d_{18}^- - d_{18}^+ = 1 \quad (\text{Mustard}) \\ \mu_{19} : & 0.4408 l_{83} - 9.9987 + d_{19}^- - d_{19}^+ = 1 \quad (\text{Poteto}) \\ \mu_{20} : & 0.0704 l_{93} - 2.7476 + d_{20}^- - d_{20}^+ = 1 \quad (\text{Pulses}) \end{aligned} \quad (11)$$

**Profit achievement goal:**

$$\begin{aligned} \mu_{21} : & 0.0123 l_{11} + 0.0916 l_{21} + 0.0198 l_{31} + 0.0265 l_{42} + 0.0222 l_{53} + 0.0159 l_{63} + 0.0078 l_{73} \\ & + 0.08364 l_{83} + 0.0135 l_{93} - 8.9999 + d_{21}^- - d_{21}^+ = 1 \end{aligned} \quad (12)$$

Now, using the data in the Table 4 and following the procedure, the deterministic equivalent of the defined chance constraints can be obtained by using the expression (5).

**Water-supply constraints:**

$$\begin{aligned} 20 l_{11} + 60 l_{21} + 34 l_{31} & \geq 4576.99, & 50 l_{42} & \geq 6318.53 \\ 7 l_{53} + 15 l_{63} + 10 l_{73} + 18 l_{83} + 10 l_{93} & \geq 1289172, \end{aligned} \quad (13)$$

where the satisfaction of probability levels are taken 0.70, 0.80, 0.90, respectively.

**Production-ratio constraint:**

The ratio of the two crops rice and wheat are considered here as the major agricultural products.

The production ratio constraint appears as:

$$(2.256 l_{31} + 2.154 l_{42} + 3.483 l_{53}) / (2.188 l_{63}) \geq 7.851, \quad (14)$$

where the satisfaction of the probability level is considered 0.90.

**Profit-ratio constraint:**

The profit ratio for Jute and Aus-paddy in the pre-kharif season is taken into account here.

The profit-ratio constraint takes the form:

$$(100.32l_{11}) / (969.45l_{21} + 161.25l_{31}) \geq 3.448, \tag{15}$$

where the probability of satisfaction of the profit ratio constraint is taken as 0.70.

Now, the executable FGP model under the four assigned priorities appears as:

Find  $\{l_{cs} \mid c = 1, 2, \dots, 9; s = 1, 2, 3\}$  so as to:

$$\begin{aligned} \text{Minimize } Z = & [P_1(0.014 d_{14}^- + 0.06 d_{15}^- + 0.088 d_{16}^- + 0.048 d_{17}^- + 0.14 d_{18}^- + 0.028 d_{19}^- + 0.26 d_{20}^-), \\ & P_2(0.027 d_1^- + 0.027 d_2^- + 0.027 d_3^-), P_3(0.012 d_4^- + 0.025 d_5^- + 0.014 d_6^- + 0.2 d_7^- + 0.09 d_8^- \\ & + 0.06 d_9^- + 3.7 d_{10}^- + 1.2 d_{11}^- + 0.65 d_{12}^-), P_4(0.00012 d_{13}^- + 0.0001 d_{21}^-)] \end{aligned} \tag{16}$$

and satisfy the membership goals in (6)-(12), subject to the system constraints in (13) – (14).

The model solution for goal achievement is presented in the Table 5.

**Table 5.** Land allocation and crops - production under the proposed model.

Crop(c)	Jute	Sugar cane	Rice	Wheat	Mustard	Potato	Pulses
Land allocation	117.06	1.76	398.24	57.71	109.02	24.95	53.23
Production	304.68	123.54	824.51	126.26	94.76	126.26	38.63

The existing and allocation and production plan is presented in the Table 6.

**Table 6.** Existing land allocation and crops - production of the year 2006-07.

Crop(c)	Jute	Sugarcane	Rice	Wheat	Mustard	Potato	Pulses
Land allocation	130.5	1.1	250.3	46.9	79.6	5.7	39.0
Production	339.66	77.4	677.7	102.6	69.1	120.2	28.3

A comparison shows that a better cropping plan is obtained here from the view point of achieving the goal levels of crops-production on the basis of the needs and desires of the DM in the decision making environment.

**Conclusion**

In the framework of proposed approach, the other different parameters (fuzzy / probabilistic) can easily be incorporated without involving any computational difficulty. In future studies, the proposed approach can be extended to cropping plan problems having the fuzzy satisficing probability levels of the chance constraints in the decision situation. Finally, it is hoped that the solution concept presented here can contribute to future studies in farming and other stochastic MODM problems in the current uncertain decision making arena.

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