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ODD GRACEFUL LABELING

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Abstract:

Graph theory studies the properties of various graphs. Graphs can be used to model many situations in the real world .Graph theory has proven to be particularly useful to a large number of rather diverse fields. The main importance of the computer, there has been a significant movement away from the traditional calculus courses and toward courses on discrete mathematics, including graph theory. We begin with simple, finite, connected and undirected graph.

KEYWORDS:
Odd graceful, Labeling, Vertex, Connected graph, Path

1.1 Introduction:
We begin with simple, finite, connected and undirected graph
$G=(V(G),E(G))$ with p vertices and q edges. For standard terminology and notations we follow Harary.

1.2 Definition:
If the vertices are assigned values subject to certain conditions then it is known as graph labeling.

1.3 Definition:
A function f is called graceful labeling of graph G if f :V {0, 1, ,… q} is injective and the induced function f^*: E {1, 2, …,q} defined as
$f^*(e = uv)= |f(u)-f(v)|$ is bijective. A graph which admits graceful labeling is called a graceful graph.

1.4 Definition:
A graph G=(V(G),E(G)) with p vertices and q edges is said to admit an odd graceful labeling if f : V (G) {0, 1,2 ,… 2q-1} is injective and the induced function f* : E(G) {1, 3,5,…,2q-1} defined as f*(e =uv) =| f(u) - f(v)| is bijective. A graph which admits graceful labelingis called an odd graceful graph.

1.5 Definition:
Shadow graph $D2(G)$ of a connected graph G is constructed by taking two copies of G say $G'$and G'', join each vertex u'in $G'$to the neighbors of the corresponding vertex u''in G''.

Theorem 1.6:

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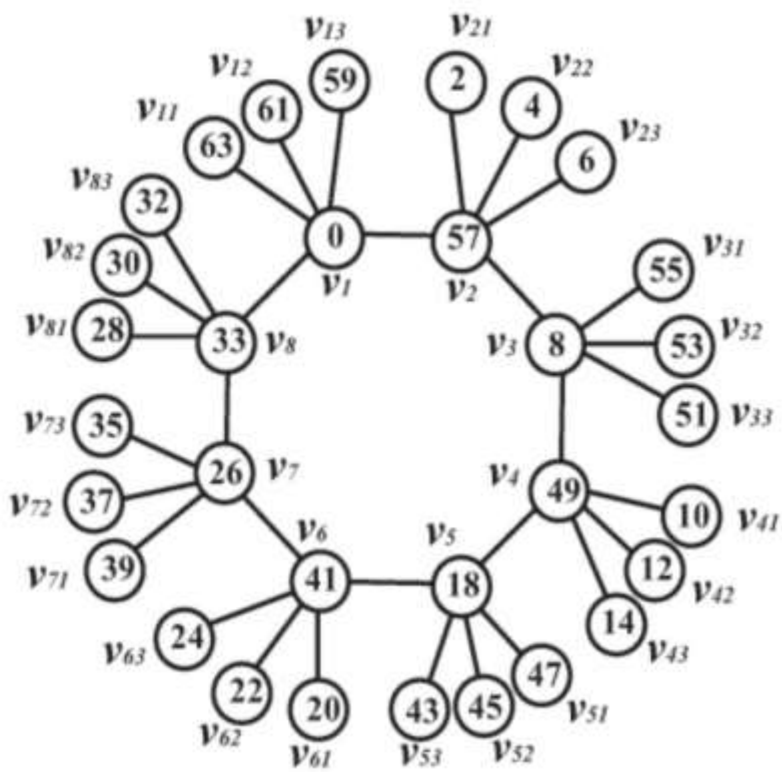
The graph obtained by fusing all the n vertices of cycle Cnof even order with the apex vertices of n copies of K1,m admits odd graceful labeling.

Proof:

Let $C_n$be a cycle of even order with $v_1, v_2, \ldots, v_n$be its vertices and G be the graph obtained by fusing all the n vertices v_i of Cnwith the apex vertices of star $K_{1,m}$.

Denote the pendant vertices of $K_{1,m}$by v_{ij} where $1 \leq i \leq n$ and $1 \leq j \leq m$.

Then G is a graph with |V (G)| = n + nm and |E(G)| = n + nm.



To define $f : V(G) \rightarrow \{0, 1, 2, \ldots. 2q - 1\}$.

we consider following two cases.

Case 1: n 0 (mod4)

$f(v_i) = (m + 1)(2n - i) + 1$; i is even and for $1 \leq i \leq$

$= (m + 1)(i - 1)$; i is odd and for $1 \leq i \leq$

$f(v_i) = (m + 1)(2n - i) + 1$; i is even and for

$+ 1 \leq i \leq n$

$= (m + 1)(i - 3) + 2(m + 2)$; i is odd and for

$+ 1 \leq i \leq n$
$f(vij)=(m + 1)(2n - i+ 1) - 2j + 1$; if i is odd
$=(m + 1)(i - 2) + 2j$; if i is even
$f(vij)=(m + 1)(2n - i+ 1) - 2j + 1$; if i is odd and For
$+ 1 \leq i\leq n; 1 \leq j \leq m$
$=(m + 1)(i - 4) + 2(m + j + 2)$; if i is even and For
$+ 1 \leq i\leq n; 1 \leq j \leq m$

Case 2: n 2(mod4)

$f(vi) = (m + 1)(2n - i) + 1$; i is even and for $1 \leq i\leq$
$+ 1$
$=(m + 1)(i - 1)$; i is odd and for $1 \leq i\leq$
$+ 1$
$f(vi) = (m + 1)(2n - i) + 1$; i is even and for
$+ 1\leq i \leq n-1$
$=(m + 1)(i - 3) + 2(m + 2)$; i is odd and for
$+ 1\leq i \leq n-1$
$f(vn) = (m + 1)(2n - i) - 1$
$f(vij)=(m + 1)(2n - i+ 1) - 2j + 1$; if i is odd and for $1 \leq i\leq$
$; 1 \leq j \leq m$
$=(m + 1)(i - 2) + 2j$; if i is even and for $1 \leq i\leq$
$; 1 \leq j \leq m$
$f(vij)=(m + 1)(2n - i+ 1) - 2j + 1$; if i is odd and for
$+ 1\leq i\leq n - 1; 1 \leq j \leq m$
$=(m + 1)(i - 2) + 2(j + 1)$; if , i is even and for
$+ 1\leq i\leq n - 1; 1 \leq j \leq m$
$f(vnj)=(m + 1)(n - 2) + 2(j + 1)$; for $1\leq j \leq m - 1$
$=(m + 1)(n -2) + 2(2j + 1)$; for $j = m$

The above defined function f exhausts all the possibilities and the graph under consideration is an odd graceful graph.

Illustration 1.7

The following Figure 1.6 shows the labeling pattern of the graph obtained by fusing each vertex of $C8$ with the apex vertices of eight copies of star $K1,3$.

Figure 1.6: Apex vertices of eight copies of star K1,3

Theorem 1.8:

The graph D2(Pn) is an odd graceful graph.

Proof:

Let G be $D2(Pn)$ then $|V(G)| = 2n$ and $|E(G)| = 4(n - 1)$ and
Let $v1, v2, \ldots, vn$ be the vertices of first copy of path Pn and $v'1, v'2, \ldots, v'n$ be the vertices of the second copy of path Pn.
Define $f : V(G)$ □ $\{0, 1, 2, \ldots, 2q - 1\}$ as follows.

$f(vi) = 4(i - 1)$; i is odd
$\quad = 4(2n - i) - 1$; i is even
$f(v'i) = 4(i - 1) + 2$; i is odd
$\quad = 4(2n - i) - 5$; i is even

The above defined function f provides graceful labeling for $D2(Pn)$.

Theorem 1.9:
The graph D2(K1,n) is an odd graceful graph.
Proof:
Let G be $D2(K1,n)$ and $v, v1, v2, \ldots, vn$ be the vertices of first copy of star $K1,n$ And $v', v'1, v'2, \ldots, v'n$ be the vertices of the second copy of star $K1,n$.
Define $f : V(G) \rightarrow \{0,1,2,\ldots(2q-1)\}$
as follows.

$f(v) = 0$
$f(vi) = 8n - 4i + 3$; for $1 \leq i \leq n$
$f(v') = 2$
$f(v'i) = 4i - 1$; for $1 \leq i \leq n$

In view of above defined labeling pattern G admits odd graceful labeling.

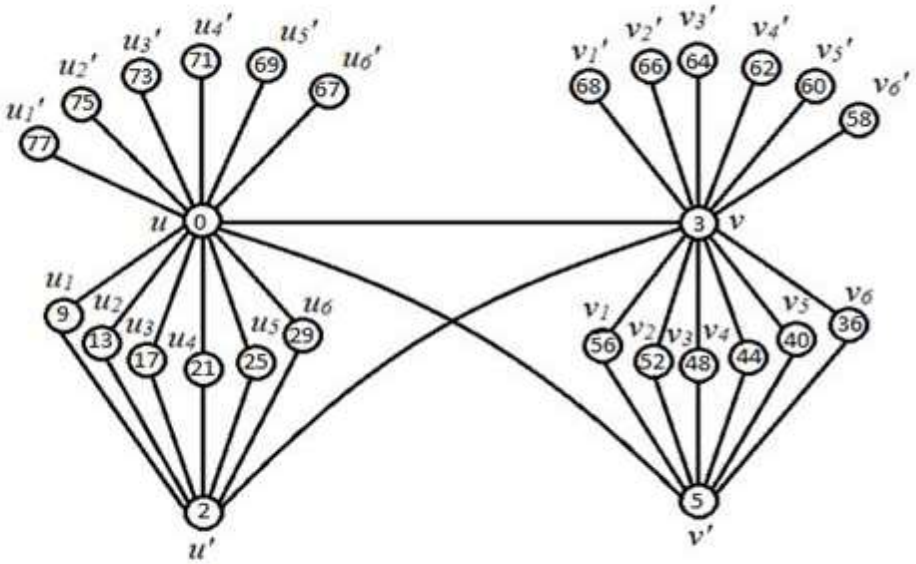
Theorem:1.10
$S'(B_{n,n})$ is an Odd graceful graph.
Proof:
Consider $B_{n,n}$ with the vertex set $\{u, v, u_i, v_i : 1 \leq i \leq n\}$, where u_i, v_i are the pendant vertices.
In order to obtain $G = S'(B_{n,n})$, add u', v', u_i', v_i' vertices corresponding to u, v, u_i, v_i; .
Where , $1 \leq i \leq n$, If $G = S'(B_{n,n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 3(2n+1)$. We define the vertex
Labeling $f : V(G) \rightarrow \{0,1,2,\ldots(12n+5)\}$
we consider the following two cases.

$f(u) = 0$,
$f(v) = 3$,
$f(u') = 2$,
$f(v') = 5$,
$f(u'i) = 5 + 4i$; $1 \leq i \leq n$
$f(u'i) = 12n + 7 - 2i$; $1 \leq i \leq n$
$f(v'1) = f(u'n) + 1$,

$f(v'1+i) = f(v'1) - 2i;$ $1 \le i \le n - 1$
$f(v1) = f(v'n) - 2,$
$f(v1+i) = f(v1) - 4i;$ $1 \le i \le n - 1$
The vertex function f defined above induces a bijective edge function
$f^*: E(G) \rightarrow \{1,3,5,...(12n+5)\}$

Thus f is an odd graceful labeling of G=$S'(B_{n,n})$.
Hence $S'(B_{n,n})$ is an odd graceful graph.

Illustration 1.11: Odd graceful labeling of the graph S'(B6,6) is shown in Figure 1.10. Figure



1.10: Odd graceful labeling of S'($B_{6,6}$).

CONCLUSION

The vertices are assigned values subject to certain conditions then it is
Known as graph labeling. The graph obtained by fusing all the n vertices of cycle
n copies of K1,m admits odd graceful labeling. D2(K1,n) ,$S'(B_{n,n})$ and D2(Pn) are Odd graceful graph

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