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Generalization Of Relation Between The Group Theory And Term Of Recurrence Relation Sequence

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Abstract

In pure mathematics number theory is very important subject . In number theory, we study about numbers and very special properties of these numbers and group theory is also very important part of pure mathematics in group theory we study about properties of groups. In this paper we define a special relation between recurrence sequence and property of group. In number theory there are many special type of recurrence relation sequence which are depend on given initial terms and recurrence relations . A recurrence relation is important topic of mathematics. Recurrence relations are used in mathematics as well as economics,physics and others subjects. My main area of research is Number Theory. Number Theory also known as Higher Arithmetic. Thinking of famous mathematician Carl Friedrich Gauss (1777– 1855) about number theory: “Mathematics is the queen of all sciences, and Number Theory is the queen of Mathematics.” Number theory or higher arithmetic is the study of those properties of integers and rational numbers, which go beyond the ordinary manipulations of everyday arithmetic.

In Number Theory [3,9] we work on numbers in mathematics many types of numbers for examples Even number, Odd number, prime number, complete square number etc. In Number Theory we want a solution in integers [10, 11]. Many basic theorems have proved in Number Theory. For example, every number can be written as product of primes.

Keywords
Sequence ,Semi Group, Term, recurrence relation, number theory, coefficient property, group.

Introduction
In group theory we study about properties of group. There are four properties of group.

1. Closure: If a and b are two elements in G, then the $a * b$ is also in G.

2. Associativity: The defined multiplication is associative, i.e., for all , a, b and c in G then

$$(a * b) * c = a * (b * c)$$

3. Identity: There exist an element e in G such that $a * e = a = e * a$ for all a belongs to G

4. Inverse: Let a is any element of G then there exist an element b in G such that

$$a * b = e = b * a$$

If G satisfied above four properties then G called the Group with respect to operation $*$ and if G satisfies the above two properties closure and associative then G becomes the Semi-Group.
In Number Theory there are many special types of Sequences Fibonacci Sequence and Luca Sequence both are special type of recurrence relation with given initial terms.[12] Recurrence relation is very useful topic of mathematics. It is an equation that defines a sequence based on a method that gives the next term as relation of the previous terms [4,7,8]. Recurrence relation is very useful in mathematics as well as economics. We can calculate growth in economics by recurrence techniques.. In mathematics, a recurrence relation is an equation that defines a sequence recursively, each term of the sequence is defined as a function of the preceding terms. **Recurrence relations** are also of fundamental importance in analysis of **algorithms**. If an **algorithm** is designed so that it will break a problem into smaller sub problems, its running time is described by a **recurrence relation.**

Number theory is the study of the set of positive whole numbers $1, 2, 3, 4, 5, 6, 7, \ldots$

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Which are often called the set of natural numbers. We will especially want to study the relationships between different sorts of numbers. Since ancient times, people have separated the natural numbers into a variety of different types.

Preliminaries

Many papers have contributed to method of solving Recurrence relations such as [1, 5 and 6]. We classify recurrence relations the number of previous terms needed to find the new term.

First Order Recurrence Relation

In the first order recurrence relation only one initial term is given. For example

$a_{n+1} = a_n + 5, n \geq 1, a_0 = 0$

we can find the terms

$a_1 = 6, a_2 = 7, a_3 = 8$

Second Order Recurrence Relation

In the second order recurrence relation new term depend on two previous terms and two initial terms are given.
For example

$$a_n = a_{n-1} + 2a_{n-2}, n \geq 2$$

with the initial terms $a_0 = 0, a_1 = 1$

Third Order Recurrence Relation

In the 3rd order recurrence relation new term is depend on previous three terms. For example

$a_n = a_{n-3} + 2a_{n-2} + a_{n-1}, n \geq 3$
with the initials terms $a_0 = 0, a_1 = 1, a_2 = 2$.

Main theorem of paper

Theorem: - let $a_{n+1} = a_n + 5, n \geq 1, a_0 = 0$ any first order recurrence sequence we get all terms of this sequence as natural number but if we takes (x, y) any real numbers which are satisfied the $x - y = 5$ $(recurrence\ relation\ of\ the\ above\ sequence)$

Let G be the collection of all (x, y) and including (0, 0) where x and y are real numbers satisfied the $x - y = 5$ then G form a semi group with respect to the operation $(x, y) * (z, t) = c_1(x, y) + c_2(z, t)$ for all $(x, y), (z, t)\ belong\ to\ G$ such that $c_1 + c_2 = 1$

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Proof

Closure property: - let $(x,y),(z,t)$ $are\ any\ two\ elements\ of\ G$ and $(x,y)*(z,t) = c_1(x,y) + c_1(z,t) = (c_1x + c_1z, c_2y + c_2t)$

Consider $(c_1x + c_2z) - (c_1y + c_2t) = c_1(x-y) + c_2(z-t) = 5c_1 + 5c_2 = 5(c_1 + c_2) = 5$

So

$$(x,y)*(z,t)\ also\ satisfied\ the\ recurrence\ relation\ i.e\ is\ also\ elements\ of\ G$$

Associative property: - these are real number so must be satisfied the associative property

So we can say that is the semi group with respect to this operation.

Conclusion

In this paper we give the special relation between semi group and recurrence relation sequence and semi group. A recurrence relation sequence is the topic of number theory and semi group is the topic of group theory. We define a relation between two branch of mathematics number theory and group theory. Recurrence relation is very useful topic of mathematics many problems of real life many be solved by recurrence relations. In this paper we are try relates group theory to recurrence relation so that we can makes application of recurrence relation more useful.

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