ISSN:0971-1260
Vol-22- Issue-14-December-2019

Green Vendor-Buyer Fuzzy Inventory Model

A.Theresal Jeyaseeli and W. Ritha

PG and Research Department of Mathematics ,

Holy Cross college (Autonomous),

Tiruchirappalli- 620002 , India.

Abstract:

Supply chain and logistics activities are the most important sources of carbon emission and environmental pollutions. These issues have increased concerns to reduce carbon dioxide emissions amount through the development of an inventory model. A green inventory model focuses on carbon cap-and-trade mechanism which is used to lower carbon emission. This paper develops fuzzy optimal total cost function and fuzzy production quantity using trapezoidal numbers and applying Lagrangian method. A numerical example follows to justify the solution procedure.

Keywords: Green inventory, Fuzzy number, Graded mean integration, Lagrangian method.

1. Introduction

Business activities can pose a major threat to the environment in terms of carbon monoxide emissions, discarded packaging materials, scrapped toxic materials, traffic congestion and other forms of industries pollution. Research on carbon emission management is becoming a significance part of the green supply chain as more business maintain to make it part of their business strategy, amid pressures from customer, competitors and regulatory agencies. Carbon emission trading is efficient way to diminish carbon emission in which mandatory limit or cap is set on carbon emission over a specific time period and a total number of permits or carbon credits are allocated to firms by regulating institutions. Green manufacturing is a production process which converts into output by reducing hazardous substances, increasing energy efficiency in lighting and heating, minimizing waste, actively designing and redesigning green processes.

The economic order quantity (EOQ) can be traced back to Harris. Benjaafar et al and chen et al developed EOQ like models, subject to the quantity of carbon emission. Their findings provided insight on business decisions while considered the minimization of carbon emission. Hua et al presented a mathematical model that includes the carbon emission cost through a cap-and-trade system under the assumption that carbon emission from logistics and warehouse is linear with order quantity. The authors also investigated the impacts of carbon cap-and-trade system on operational order decisions, carbon emissions and total cost.

Inspired by the above mentioned works in this paper, we have considered trapezoidal fuzzy numbers and optimization method is carried out by Lagrangian method. For defuzzifying the total cost graded mean integration method has been applied. Finally, numerical example is illustrated.

ISSN:0971-1260
Vol-22- Issue-14-December-2019

2. Preliminaries

2.1 Definitions

Definition 2.1.1

A fuzzy set $\tilde{a}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{a}}(x)$ that maps each element x in X to a real number in the interval [0,1]. The function value $\mu_{\tilde{a}}(x)$ is termed the grade of membership of x in $\tilde{a}$. The nearer the value of $\mu_{\tilde{a}}(x)$ to unity, the higher the grade of membership of x in $\tilde{a}$.

Definition 2.1.2

A fuzzy set $\tilde{A}$ defined on the set of real numbers R, is said to be a **fuzzy number** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ as the following characteristics,

(i) $\tilde{A}$ is fuzzy convex

(ii) $\tilde{A}$ is normal

(iii) $\tilde{A}$is piecewise continuous

Definition 2.1.3

A fuzzy set $\tilde{A}$, membership function $\mu_{\tilde{A}}$ that maps each element in X to a real number in the interval [0,1]. A **trapezoidal fuzzy number** is represented by$\tilde{A} = \{a_1, a_2, a_3, a_4\}$, where a_1, a_2, a_3, a_4 are real numbers and its membership function is given below.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for} \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{for} \quad a_1 \le x \le a_2 \\ 1 & \text{for} \quad a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{for} \quad a_3 \le x \le a_4 \\ 0 & \text{for} \quad x > a_4 \end{cases}$$

2.2 Extension of the Lagrangian method

Solving a nonlinear programming problem by obtaining the optimum solution was discussed by Taha with equality constraints, also by solving those inequality constraints using Lagrangian method.

Suppose if the problem is given as
min y=f(x)
subject to g:(x)≥0, i=1,2,3,…m.
The constraints are non-negative say x≥0 if included in the m constraints. Then the procedure of the extension of the Lagrangian method will involve the following steps.

Step 1: Solve the unconstrained problem min y=f(x)
If the resultin optimum satisfies all the constraints, then stop since all the constraints are inessential or else set k =1 and move to step 2.

Step 2: Activate any k constraints (i.e., convert them into equalities) and optimize f(x) subject to the k active constraints by the Lagrangian method. If the resulting solution is feasible with respect to the remaining constraints, we shall repeat the steps. If all sets of

ISSN:0971-1260
Vol-22- Issue-14-December-2019

active constraints taken k at a time are considered without confront a feasible solution, go to step 3.
Step 3: If k = m, stop; there's no feasible solution, otherwise set k = k+1 and go to step 2.

2.3 Graded Mean integration representation method

Graded mean integration representation method was introduced by chen and Hsieh based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. First we define a generalized fuzzy number as follows:

$$\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$$

By graded mean integration are the inverses of L and R are L^{-1} and R^{-1} respectively. The graded mean h-level value of the generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ is given by $\frac{h}{2}[L^{-1}(h) + R^{-1}(h)]$. Then the graded mean integration representation of $P(\tilde{A})$ with grade then $P(\tilde{A}) = \dfrac{\int_0^{w_A} \frac{h}{2}[L^{-1}(h) + R^{-1}(h)]dh}{\int_0^{w_A} hdh}$ where $0 < h \le w_A$ and $0 < w_A \le 1$.

In this paper, we have used trapezoidal fuzzy numbers as fuzzy parameters for the production inventory model. Let $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then the graded mean integration representation is given by the formula as

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2}[(b_1 + b_4) + h(b_2 - b_1 - b_4 + b_3)]dh}{\int_0^1 hdh}$$

$$P(\tilde{B}) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

3. Assumptions and Notations

The model is developed on the following assumptions and notations.

3.1 Notations

D	Annual demand
Q	Order quantity per cycle
A	Order cost per cycle
C	Carbon emission price
A_C	Carbon emission quantity for order per cycle
S	Setup cost
S_C	Carbon emission from setup
n	Number of times the order is produced
M	Manufacturing cost
M_C	Carbon emission from manufacturing

ISSN:0971-1260
Vol-22- Issue-14-December-2019

C_S	Screening cost
LCA(T)	Life cycle assessment technology cost
h_r	Inventory holding cost per unit time
h_{rc}	Carbon emission quantity from inventory holding
P	Production rate
h_m	Inventory holding cost per unit time and quantity for manufacturer
h_{mc}	Carbon emission quantity from inventory holding for manufacturer
p	Labour cost for packing per parcel
L	The cost of material used for packing per parcel
N	Number of parcels
l	Fixed cost per trip
m	Variable cost per unit transported per distance travelled
d	Distance travelled
x	Proportion of demand returned
g	Social cost from vehicle emission
v	Average velocity
ψ_o	Fixed cost per waste disposal activity
ψ	Cost to dispose waste to the environment
θ	Proportion of waste produced per lot
α	Cap of carbon emission for the retailer
β	Cap of carbon emission for the manufacturing industry
R_N	Revenue earned due to LCA

3.2 Assumptions

1. Life cycle assessment (LCA) approach is used for assessing products environmental impacts.
2. The units transported are finally packed in parcels.
3. Waste management focuses on source reduction, pollution prevention and disposal.
4. The materials converted using LCA technology are reused as raw materials in manufacturing.
5. For retailer and manufacturer, the carbon emission quantities obey EOQ and EPQ assumptions, respectively.
6. Eco-friendly materials are used for packaging.

4. Mathematical Model

4.1 Crisp Model

For the retailer manufacturer system under a carbon cap-and-trade mechanism with green inventory model comprises of green procurement cost, green setup cost, green manufacturing cost, holding cost, screening cost, green distribution cost, green logistics cost, cap and trade, waste produced by the inventory system, life cycle assessment technology cost.

$$TC(\mathrm{Q}) = \frac{D}{Q}[(A + CA_C) + (\frac{S + CS_C}{n}) + (M + CM_C) + C_S + LCA(T) + 2l + \frac{2gd}{v} + \psi_o + (p + L)N]$$
$$+\frac{Q}{2}[(h_r + Ch_{rc}) + (n(1+\frac{D}{P}) - 1)(h_m + Ch_{mc})] + \frac{md(1+x) + \psi(\theta + x)}{D} - C(\alpha + \beta) - R_N$$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

$$TC(Q)=\frac{D}{Q}[A+M+(p+L)N+H]+\frac{Q}{2}[a+(n(1+\frac{D}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D}-C(\alpha+\beta)-R_N$$

Where $H=CA_C+(\frac{S+CS_C}{n})+CM_C+C_S+LCA(T)+2l+\frac{2gd}{v}+\psi_o$

$$a=h_r+Ch_{rc}$$

$$b=h_m+Ch_{mc}$$

In order to find the optimal order quantity the above equation is differential with respect to Q and equated to zero.

The optimal order quantity is derived as

$$Q^*=\sqrt{\frac{D[A+M+(p+L)N+H]}{\frac{1}{2}[a+(n(1+\frac{D}{P})-1)b]}}$$

4.2 Fuzzy Model

The total inventory cost is

$$TC(Q)=\frac{D}{Q}[A+M+(p+L)N+H]+\frac{Q}{2}[a+(n(1+\frac{D}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D}-C(\alpha+\beta)-R_N$$

Suppose

$$\tilde{Q}=(Q_1,Q_2,Q_3,Q_4)$$

$$\tilde{D}=(D_1,D_2,D_3,\mathrm{D}_4)$$

$$\tilde{A}=(\mathrm{A}_1,\mathrm{A}_2,\mathrm{A}_3,\mathrm{A}_4)$$

$$\tilde{M}=(\mathrm{M}_1,\mathrm{M}_2,\mathrm{M}_3,\mathrm{M}_4)$$

$$\tilde{p}=(p_1,p_2,p_3,p_4)$$

$$\tilde{L}=(\mathrm{L}_1,\mathrm{L}_2,\mathrm{L}_3,\mathrm{L}_4)$$

$$\begin{aligned}\mathrm{TC(Q)}=\frac{1}{6}\{&[\frac{D_1}{Q}[A_1+M_1+(p_1+L_1)N+H]+\frac{Q}{2}[a+(n(1+\frac{D_1}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_4}-C(\alpha+\beta)-R_N]\\&+2[\frac{D_2}{Q}[A_2+M_2+(p_2+L_2)N+H]+\frac{Q}{2}[a+(n(1+\frac{D_2}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_3}-C(\alpha+\beta)-R_N]\\&+2[\frac{D_3}{Q}[A_3+M_3+(p_3+L_3)N+H]+\frac{Q}{2}[a+(n(1+\frac{D_3}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_2}-C(\alpha+\beta)-R_N]\\&+[\frac{D_4}{Q}[A_4+M_4+(p_4+L_4)N+H]+\frac{Q}{2}[a+(n(1+\frac{D_4}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_1}-C(\alpha+\beta)-R_N]\}\end{aligned}$$

Partially differentiating with respect to Q and equating to zero,

$$\mathrm{Q}=\sqrt{\frac{\begin{array}{l}[D_1(A_1+M_1+p_1N+L_1N+H)+2D_2(A_2+M_2+p_2N+L_2N+H)\\+2D_3(A_3+M_3+p_3N+L_3N+H)+D_4(A_4+M_4+p_4N+L_4N+H)]\end{array}}{\begin{array}{l}[\frac{1}{2}(a+(n(1+\frac{D_1}{P})-1)b)+(a+(n(1+\frac{D_2}{P})-1)b)\\+(a+(n(1+\frac{D_3}{P})-1)b)+\frac{1}{2}(a+(n(1+\frac{D_4}{P})-1)b)]\end{array}}}$$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

$$\mathrm{TC}=\frac{1}{6}\{[\frac{D_1}{Q_4}[A_1+M_1+(p_1+L_1)N+H]+\frac{Q_1}{2}[a+(n(1+\frac{D_1}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_4}-C(\alpha+\beta)-R_N]$$
$$+2[\frac{D_2}{Q_3}[A_2+M_2+(p_2+L_2)N+H]+\frac{Q_2}{2}[a+(n(1+\frac{D_2}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_3}-C(\alpha+\beta)-R_N]$$
$$+2[\frac{D_3}{Q_2}[A_3+M_3+(p_3+L_3)N+H]+\frac{Q_3}{2}[a+(n(1+\frac{D_3}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_2}-C(\alpha+\beta)-R_N]$$
$$+[\frac{D_4}{Q_1}[A_4+M_4+(p_4+L_4)N+H]+\frac{Q_4}{2}[a+(n(1+\frac{D_4}{P})-1)b]+\frac{md(1+x)+\psi(\theta+x)}{D_1}-C(\alpha+\beta)-R_N]\}$$

Solving the non-constraint problem, now we partially differentiating with respect to Q_1, Q_2, Q_3, Q_4 respectively,

$$\frac{\partial TC}{\partial Q_1}=\frac{1}{6}[\frac{1}{2}(a+(n(1+\frac{D_1}{P})-1)b)-\frac{D_4}{Q_1^2}(A_4+M_4+(p_4+L_4)N+H)]$$

Letting $\frac{\partial TC}{\partial Q_1}=0$

$$Q_1=\sqrt{\frac{D_4(A_4+M_4+(p_4+L_4)N+H)}{\frac{1}{2}(a+(n(1+\frac{D_1}{P})-1)b)}}$$

Letting $\frac{\partial TC}{\partial Q_2}=0$

$$\frac{\partial TC}{\partial Q_2}=\frac{1}{6}[(a+(n(1+\frac{D_2}{P})-1)b)-\frac{2D_3}{Q_2^2}(A_3+M_3+(p_3+L_3)N+H)]=0$$

$$\mathrm{Q}_2=\sqrt{\frac{2D_3(A_3+M_3+(p_3+L_3)N+H)}{(a+(n(1+\frac{D_2}{P})-1)b)}}$$

$$\frac{\partial TC}{\partial Q_3}=\frac{1}{6}[(a+(n(1+\frac{D_2}{P})-1)b)-\frac{2D_2}{Q_3^2}(A_2+M_2+(p_2+L_2)N+H)]$$

Letting $\frac{\partial TC}{\partial Q_3}=0$

$$\mathrm{Q}_3=\sqrt{\frac{2D_2(A_2+M_2+(p_2+L_2)N+H)}{(a+(n(1+\frac{D_3}{P})-1)b)}}$$

$$\frac{\partial TC}{\partial Q_4}=\frac{1}{6}[\frac{1}{2}(a+(n(1+\frac{D_4}{P})-1)b)-\frac{D_1}{Q_4^2}(A_1+M_1+(p_1+L_1)N+H)]$$

Letting $\frac{\partial TC}{\partial Q_4}=0$

$$\mathrm{Q}_4=\sqrt{\frac{D_1(A_1+M_1+(p_1+L_1)N+H)}{\frac{1}{2}(a+(n(1+\frac{D_4}{P})-1)b)}}$$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

The above results showthat $Q_1 > Q_2 > Q_3 > Q_4$, but contrastingly we have $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. Hence W and set k = 1 and we convert the inequality constraint by optimizing the total cost subject to Lagrangian method subject to $Q_1 - Q_2 = 0$.

$TC(Q_1, Q_2, Q_3, Q_4, \lambda) = TC(Q) - \lambda(Q_2 - Q_1)$

Now taking the partial derivatives with respect to Q_1, Q_2, Q_3, Q_4 and λ and the minimize $TC(Q_1, Q_2, Q_3, Q_4, \lambda)$, we have

$$\frac{\partial TC}{\partial Q_1} = 0$$

$$\frac{\partial TC}{\partial Q_1} = \frac{1}{6}[\frac{1}{2}(a + (n(1 + \frac{D_1}{P}) - 1)b) - \frac{D_4}{Q_1^2}(A_4 + M_4 + (p_4 + L_4)N + H)] + \lambda = 0$$

$$\frac{\partial TC}{\partial Q_2} = 0$$

$$\frac{\partial TC}{\partial Q_2} = \frac{1}{6}[(a + (n(1 + \frac{D_2}{P}) - 1)b) - \frac{2D_3}{Q_2^2}(A_3 + M_3 + (p_3 + L_3)N + H)] - \lambda = 0$$

$$\frac{\partial TC}{\partial Q_3} = 0$$

$$\frac{\partial TC}{\partial Q_3} = \frac{1}{6}[(a + (n(1 + \frac{D_2}{P}) - 1)b) - \frac{2D_2}{Q_3^2}(A_2 + M_2 + (p_2 + L_2)N + H)] = 0$$

$$\frac{\partial TC}{\partial Q_4} = 0$$

$$\frac{\partial TC}{\partial Q_4} = \frac{1}{6}[\frac{1}{2}(a + (n(1 + \frac{D_4}{P}) - 1)b) - \frac{D_1}{Q_4^2}(A_1 + M_1 + (p_1 + L_1)N + H)] = 0$$

$$\frac{\partial TC}{\partial \lambda} = -(Q_2 - Q_1)$$

$$Q_1 = Q_2 = \sqrt{\frac{D_4(A_4 + M_4 + (p_4 + L_4)N + H) + 2D_3(A_3 + M_3 + (p_3 + L_3)N + H)}{\frac{1}{2}(a + (n(1 + \frac{D_1}{P}) - 1)b) + (a + (n(1 + \frac{D_2}{P}) - 1)b)}}$$

$$Q_3 = \sqrt{\frac{2D_2(A_2 + M_2 + (p_2 + L_2)N + H)}{(a + (n(1 + \frac{D_3}{P}) - 1)b)}}$$

$$Q_4 = \sqrt{\frac{D_1(A_1 + M_1 + (p_1 + L_1)N + H)}{\frac{1}{2}(a + (n(1 + \frac{D_4}{P}) - 1)b)}}$$

Now converting the inequality constraints $Q_2 - Q_1 \ge 0$ and $Q_3 - Q_2 \ge 0$ into equality constraints $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0$.

Optimizing $TC(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2) = TC(Q) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2)$

$$\frac{\partial TC}{\partial Q_1} = 0$$

$$\frac{\partial TC}{\partial Q_1} = \frac{1}{6}[\frac{1}{2}(a + (n(1 + \frac{D_1}{P}) - 1)b) - \frac{D_4}{Q_1^2}(A_4 + M_4 + (p_4 + L_4)N + H)] + \lambda_1 = 0$$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

$$\frac{\partial TC}{\partial Q_2}=0$$

$$\frac{\partial TC}{\partial Q_2}=0\frac{\partial TC}{\partial Q_2}=\frac{1}{6}[(a+(n(1+\frac{D_2}{P})-1)b)-\frac{2D_3}{Q_2^{\ 2}}(A_3+M_3+(p_3+L_3)N+H)]-\lambda_1+\lambda_2=0$$

$$\frac{\partial TC}{\partial Q_3}=0$$

$$\frac{\partial TC}{\partial Q_3}=\frac{1}{6}[(a+(n(1+\frac{D_2}{P})-1)b)-\frac{2D_2}{Q_3^{\ 2}}(A_2+M_2+(p_2+L_2)N+H)]-\lambda_2=0$$

$$\frac{\partial TC}{\partial Q_4}=0$$

$$\frac{\partial TC}{\partial Q_4}=\frac{1}{6}[\frac{1}{2}(a+(n(1+\frac{D_4}{P})-1)b)-\frac{D_1}{Q_4^{\ 2}}(A_1+M_1+(p_1+L_1)N+H)]=0$$

$$Q_1=Q_2=Q_3=\sqrt{\frac{D_4(A_4+M_4+(p_4+L_4)N+H)+2D_3(A_3+M_3+(p_3+L_3)N+H)+2D_2(A_2+M_2+(p_2+L_2)N+H)}{\frac{1}{2}(a+(n(1+\frac{D_1}{P})-1)b)+(a+(n(1+\frac{D_2}{P})-1)b)+(a+(n(1+\frac{D_3}{P})-1)b)}}$$

$$\mathrm{Q}_4=\sqrt{\frac{D_1(A_1+M_1+(p_1+L_1)N+H)}{\frac{1}{2}(a+(n(1+\frac{D_4}{P})-1)b)}}$$

Again converting the inequality constraints $Q_2-Q_1\geq 0$, $Q_3-Q_2\geq 0$ and $Q_4-Q_3\geq 0$ into equality constraints by $Q_2-Q_1=0$, $Q_3-Q_2=0$ and $Q_4-Q_3=0$ using Lagrangian method. The Lagrangian function is given by

$$TC(Q_1,Q_2,Q_3,Q_4,\lambda_1,\lambda_2,\lambda_3)=TC(Q)-\lambda_1(Q_2-Q_1)-\lambda_2(Q_3-Q_2)-\lambda_3(Q_4-Q_3)$$

$$\frac{\partial TC}{\partial Q_1}=0$$

$$\frac{\partial TC}{\partial Q_1}=\frac{1}{6}[\frac{1}{2}(a+(n(1+\frac{D_1}{P})-1)b)-\frac{D_4}{Q_1^{\ 2}}(A_4+M_4+(p_4+L_4)N+H)]+\lambda_1=0$$

$$\frac{\partial TC}{\partial Q_2}=0$$

$$\frac{\partial TC}{\partial Q_2}=\frac{1}{6}[(a+(n(1+\frac{D_2}{P})-1)b)-\frac{2D_3}{Q_2^{\ 2}}(A_3+M_3+(p_3+L_3)N+H)]-\lambda_1+\lambda_2=0$$

$$\frac{\partial TC}{\partial Q_3}=0$$

$$\frac{\partial TC}{\partial Q_3}=\frac{1}{6}[(a+(n(1+\frac{D_2}{P})-1)b)-\frac{2D_2}{Q_3^{\ 2}}(A_2+M_2+(p_2+L_2)N+H)]-\lambda_2+\lambda_3=0$$

$$\frac{\partial TC}{\partial Q_4}=0$$

$$\frac{\partial TC}{\partial Q_4}=\frac{1}{6}[\frac{1}{2}(a+(n(1+\frac{D_4}{P})-1)b)-\frac{D_1}{Q_4^{\ 2}}(A_1+M_1+(p_1+L_1)N+H)]-\lambda_3=0$$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

$$\frac{\partial TC}{\partial \lambda_1} = -(Q_2 - Q_1)$$
$$\frac{\partial TC}{\partial \lambda_2} = -(Q_3 - Q_2)$$
$$\frac{\partial TC}{\partial \lambda_3} = -(Q_4 - Q_3)$$

$$Q_1 = Q_2 = Q_3 = Q_4 = \sqrt{\frac{\begin{array}{l} D_4(A_4 + M_4 + (p_4 + L_4)N + H) + 2D_3(A_3 + M_3 + (p_3 + L_3)N + H) \\ +2D_2(A_2 + M_2 + (p_2 + L_2)N + H) + D_1(A_1 + M_1 + (p_1 + L_1)N + H) \end{array}}{\begin{array}{l} \frac{1}{2}(a + (n(1+\frac{D_1}{P}) - 1)b) + (a + (n(1+\frac{D_2}{P}) - 1)b) \\ +(a + (n(1+\frac{D_3}{P}) - 1)b) + \frac{1}{2}(a + (n(1+\frac{D_4}{P}) - 1)b) \end{array}}}$$

Therefore $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ satisfies all the inequality constraints and we obtain the optimum solution for the problems.

Let $Q_1 = Q_2 = Q_3 = Q_4 = \tilde{Q}^*$. Then optimal fuzzy EOQ is given by

$$\tilde{Q}^* = \sqrt{\frac{\begin{array}{l} D_4(A_4 + M_4 + (p_4 + L_4)N + H) + 2D_3(A_3 + M_3 + (p_3 + L_3)N + H) \\ +2D_2(A_2 + M_2 + (p_2 + L_2)N + H) + D_1(A_1 + M_1 + (p_1 + L_1)N + H) \end{array}}{\begin{array}{l} \frac{1}{2}(a + (n(1+\frac{D_1}{P}) - 1)b) + (a + (n(1+\frac{D_2}{P}) - 1)b) \\ +(a + (n(1+\frac{D_3}{P}) - 1)b) + \frac{1}{2}(a + (n(1+\frac{D_4}{P}) - 1)b) \end{array}}}$$

5. Numerical Illustration

Consider the following data

D	60000	C_S	0.2	x	0.2
P	80000	LCA(T)	0.25	g	0.5
A	200	h_r	0.8	V	180
C	0.2	h_{rc}	2	ψ_o	1
A_C	400	h_{mc}	1	ψ	0.8
S	800	L	2	θ	0.5
S_C	400	N	600	α	8000
n	1	l	50	β	6000
M	300	m	5	R_N	200
M_C	400	d	250	h_m	0.4
p	0.3				

Crisp model:
The optimal order quantity Q^* =12069 units
Fuzzy model:

ISSN:0971-1260
Vol-22- Issue-14-December-2019

The optimal order quantity $\tilde{Q}^*$ =14648 units

6. Conclusion:

In this paper a green inventory model with cap and trade mechanism is presented. The notations are taken as trapezoidal fuzzy numbers, Lagrangian method has been applied and defuzzification has been done by graded mean representation method. The firm and society are benefited by using this green inventory model. This model helps the firm to improve productivity since it uses natural light and ventilation. This model helps in creating healthier environments which leads to less toxins and cleaner air with less hazardous production processes.

REFERENCES:

[1] Beamon BM., "Designing the green supply chain", Logistics Information Management 1999; 12(4): 332-342.

[2] Chen S.H and Hsieh C.H., "Graded mean integration representations of generalized fuzzy number", Journal of Chinese Fuzzy Systems, 5(1999), 1-7.

[3] Chin T.A., Tat H.H., and Sulaiman Z., (2015), "Green supply chain management", Environmental Collaboration and Sustainability Performance, Procedia CIRP, 26,695-699.

[4] Green, Zelbst PJ, Meacham J, Bhadauria V S, "Green supply chain management practices; impact on performance", Supply Chain Management: An International Journal 2012; 17/3: 290-305.

[5] Hadley G., and Whitin T.M., "Analysis of Inventory systems", Prentice Hall, Englewood clipps, NJ, 1963.

[6] Holt D, Ghobadian A., "An empirical study of Green Supply Chain Management practices amongst UK manufacturers", Journal of Manufacturing Technology Management 2009; 20(7): 933-956.

[7] Li Y., "Research on the Performance Measurement of Green Supply Chain Management in china", Journal of Sustainable Development 2011; 4(3): 101-107.

[8] Luthra S, Garg D, Haleem A., "Green Supply Chain Management", Implementation and Performances-a literature review and some issues, Journal of Advances in Management Research 2014; 11(1): 20-46.

[9] Mentzer JT., "Supply chain management", United States of America, Sage Publications, Inc; 2001.

[10] Ninlawan C, Seksan P, Tossapol K, Pilada W., "The implementation of green supply chain management practices in electronics industry", Proceedings of the International Multi conferences of Engineers and Computer Scientists 2010; March 17-19: Hong Kong.

[11] Sarkis J. and Tamarkin M., (2005). "Real options analysis for green trading: The case of green house gases",The engineering Economist, 50(3), 273-294.

[12] Taft E. (1918), "The most economical Production lot", The Iron Age, 101,1410-1412.

[13] Tao Z.,(2013), "Carbon Emission Modeling in Green Supply Chain Management", Retrieved from https://etd.ohiolink.edu/

[14] Tao Z., Guiffrida A.L and Troutt M.D (2012), "A Green Cost based economic production / order quantity model", Proceeding of the 1st International Symposium on Green Supply Chains, Arkon, OH, July 30, 2010.

[15] Wisner JD, Tan K-C, Leong GK, "Supply Chain Management: a balanced approach", 3rd ed. Canada: South-Western Cengage Learning; 2012.

