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Adaptive Backstepping Based Control of Fractional Order Lorenz System with Unknown Parameters

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***Abstract*—This paper presents a backstepping based step by step controller designer procedure for a fractional order chaotic system (FOCS) with uncertain parameters, which is in strict feedback form. On the basis of backstepping control, a control strategy for stabilization of fractional order Lorenz system with unknown parameters, is proposed. Adaptive backstepping control has been used to obtain the parameter update laws for Lorenz system with unknown parameters. The controller is obtained such that the singularity problem is avoided, simultaneously, getting the update laws for the parameters. Simulation results are presented to prove the effectiveness of thecontroller.**

***Keywords*—fractional order; backstepping; uncertainty; Lorenz system**

I. Introduction

Chaos that is considered one among the necessary properties of nonlinear systems, finds varied applications in engineering and science. A lot of researchers have focused on chaos control [1], [2], and further towards chaos synchronization [3]–[5]. Fractional calculus can assist in obtaining the mathematical representation of a system, more accuratelyas compared to the traditionalmodelling methods, which leads to better analysis and control[6], [7]. FOCShave become one of the important fileds of research.

Fractional order variants of different integer-order chaotic systems have been studied and analyzed e.g.,Lorenz system [8], Chen system [9], [10],Rössler's system [11]. Vrious techniques have been put forward for synchronizationof FOCS[12]–[17]. Backstepping technique is one of the prevalent controller design method put forward byKristic et. al. [18]. It has been used by severalscholars for control and synchronization of numerous systems.Backstepping procedure is based on Lyapunov theory and involves step-wisetechnique for controller strategy. It also warrantsasymptotic stabilityin global sense.

Various researchers have put forward different methods for stabilization of systems with unfamiliar parameters [29-31]. In this paper, we propose a stabilizing controller for the fractional order (FO) Lorenz system with unknown parameters. Here, in this manuscript, controller is designed on the basis of adaptive backstepping method, for uncertain FO Lorenz system which itself is based on FO extension of Lyapunov stability theory.Also, different techniques have been utilized for regulation of theconcerned system, but backstepping method has not been applied to address this problem. Traditional backstepping if applied results into singularity problem which further leads to system instability. Here in this work, the controller isobtained by utilizingadaptive backstepping strategy which results into adaptation laws for uncertain parameters, which further avoids the singularity problem.

The manuscript is systematizedas:Fractional calculus is discussed in Section II. System description and its behavior is discussed in section III. Design of stabilizing controllers for FO Lorenz systems is described in section IV. The outcomes obtained after simulationare given in section V. Section VI concludes the work.

II. Fundamentals of Fractional Calculus

The elementarydescription of fractional calculus through operator ${}_aD_t^q$ is expressed as,

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t (d\tau)^{-q} & q < 0 \end{cases} \quad (1)$$

Here, a and t are the limitsof integration and q is a real number.

Theimperativedefinitions which describeFO differentiation or integral are:Grunwald-Letnikov,Riemann-Liouville (RL), and definition given by Caputo.

A. Grunwald-Letnikov

$$ {}_a^{GL}D_t^q f(t) = \lim_{h \to 0} \frac{1}{h^q} \sum_{j=0}^{\infty} (-1)^j \binom{q}{j} f(t - jh) \quad (2)$$

B. Riemann-Liouville

$$ \mathcal{J}^q f(t) \triangleq \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau) d\tau \quad (3)$$

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$$ {}^{RL}_{a}D_t^q f(t) = \frac{1}{\Gamma(m-q)} \frac{d^m}{dt^m} \int_a^t \frac{f(\tau)}{(t-\tau)^{q-m+1}} d\tau $$

where, $\Gamma(m) = (m-1)!$, $t > 0$, $q \in \mathbb{R}^+ m - 1 < q < m$

C. Caputo

$$ {}^{C}_{a}D_t^q f(t) = \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{q-m+1}} f(\tau) d\tau \qquad (4) $$

The numerical approximations of thederivative of order $q$on points kh $(k = 1, 2, \ldots)$ is written as:

$$ {}_{k-L_m/h}D_{kh}^q f(t) = h^{-q} \sum_{j=0}^{k} c_j^{(q)} f(t_{k-j}) \qquad (5) $$

with, L_m = ‘memory length, h = time step. $c_j^{(q)} (j = 0,1, \ldots, k)$ are coefficients expressed as:

$$ c_0^{(q)} = 1 $$

$$ c_j^{(q)} = \left(1 - \frac{1+q}{j}\right) c_{j-1}^{(q)} \qquad (6) $$

The final solution of nonlinear FODE expressed as, ${}_aD_t^q y(t) = f(y(t), t)$, is given as:

$$ y(t_k) = f(y(t_k), t_k) h^q - \sum_{j=1}^{k} c_j^{(q)} y(t_{k-j}) \quad (7) $$

III. System Description and Stability Analysis

The Lorenz system represents a set of ordinary differential equations and was first studied by Edward Lorenz in 1963. It is considered as one of the benchmark systems by research community, in the area of nonlinear dynamics. The integer order Lorenz system is expressed as:

$$ \dot{x}_1(t) = \theta_1 x_2(t) - \theta_1 x_1(t) $$
$$ \dot{x}_2(t) = -x_1(t)x_3(t) + \theta_2 x_1(t) - x_2(t) \quad (8) $$
$$ \dot{x}_3(t) = x_1(t)x_2(t) - \theta_3 x_3(t) + u $$

The state model of FOversion of Lorenz system iswritten as:

$$ D_t^{q1} x_1(t) = \theta_1 x_2(t) - \theta_1 x_1(t) $$
$$ D_t^{q2} x_2(t) = -x_1(t)x_3(t) + \theta_2 x_1(t) - x_2(t) \quad (9) $$
$$ D_t^{q3} x_3(t) = x_1(t)x_2(t) - \theta_3 x_3(t) + u $$

where, θ_1, θ_2 and θ_3 are parameters of the system and are unknown and q_1, q_2 and q_3 are orders of derivative. The controller u is to be designed to establish the stability of the system and for estimation of uncertain parameters.

The existence of chaosin the system (9) is establishedby phase portraits given in Figures 1and 2. The initial conditions and the parameters are taken as: $(\theta_1 = 10, \theta_2 = 28, \theta_3 = 8/3)$and $(x_1(0) = 0.1, x_2(0) = 0.1, x_3(0) = 0.1)$, respectively. Thederivative orders are taken as $q_1 = q_2 = q_3 = 0.995$.

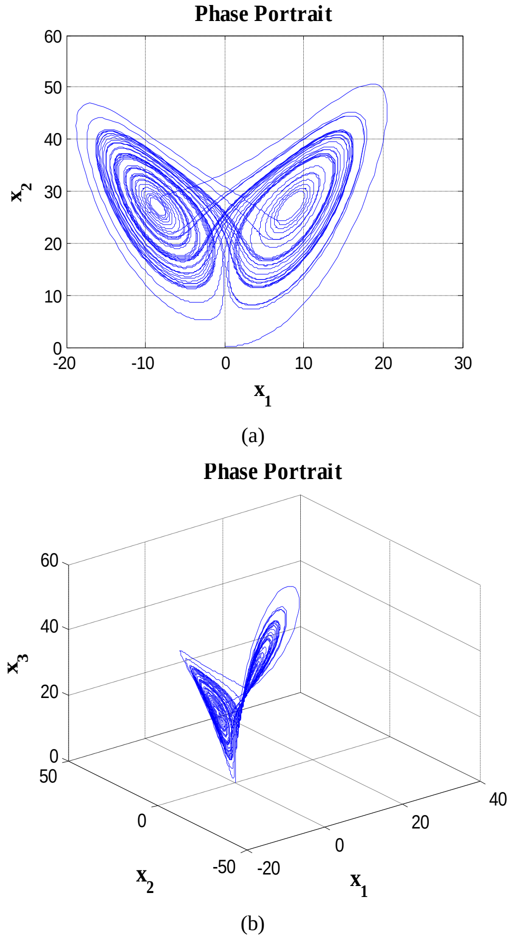

Fig.1: Chaotic behavior of FO Lorenz system (a) for states x_1 and x_2. (b) for states x_1, x_2 and x_3.

IV. Stabilization Using Backstepping Control

As the backstepping control is based on Lyapunov stability criterion, we use the extension of Lyapunov stability [19], [20]. To apply adaptive backstepping control technique we need to transform the controlled Lorenz system (9) into the general parametric strict-feedback form with $n = 3$.

$$ D_t^q x_1(t) = g_1(x_1, t)x_2 + \theta^T F_1(x_1, t) + f_1(x_1, t) $$
$$ D_t^q x_2(t) = g_2(x_1, x_2, t)x_3 + \theta^T F_2(x_1, x_2, t) + f_2(x_1, x_2, t) $$
$$ \vdots \qquad (10) $$
$$ D_t^q x_{n-1}(t) = g_{n-1}(x_1, x_2, \ldots x_{n-1}, t)x_n + \theta^T F_{n-1}(x_1, \ldots, x_{n-1}) + f_{n-1}(x_1, \ldots, x_{n-1}) $$
$$ D_t^q x_n(t) = g_n(x_1, x_2, \ldots x_n, t)u + \theta^T F_n(x_1, \ldots, x_n) + f_n(x_1, \ldots, x_n) $$

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where, $\theta \in R^p$ is theunknown constant parameters vector.The strategy for designing stabilizing controller is given below.

For the system (9), assuming $q_1 = q_2 = q_3 = q$, let $z_1 = x_1$ and $z_2 = x_2 - \alpha_1$. It gives,

$$D^q z_1 = D^q x_1 = \theta_1 x_2 - \theta_1 x_1$$
$$= \theta_1 z_2 + \theta_1 \alpha_1 - \theta_1 x_1 \quad (11)$$

Lyapunov function for (11) is, $V_1 = \frac{1}{2} z_1^2$

$$\Rightarrow D^q V_1 \le z_1 D^q z_1$$
$$\le z_1(\theta_1 x_2 - \theta_1 x_1)$$
$$\le z_1 \theta_1 z_2 + z_1 \theta_1 (\alpha_1 - x_1)$$

the virtual controller can be chosen as:

$$\alpha_1 = -c_0 z_1 \quad (12)$$

V_1, is changedto

$$\Rightarrow D^q V_1 = -c_1 z_1^2 + \theta_1 z_1 z_2, c_1 = c_0 \theta_1 + \theta_1 > 0$$

Similarly, for $z_3 = x_3 - \alpha_2$the derivative of $z_2$is expressed as

$$D^q z_2 = -x_1 z_3 - x_1 \alpha_2 + \hat{\theta}_2 x_1 - c_0 \hat{\theta}_1 x_1 - (1 - c_0 \hat{\theta}_1)\alpha_1 - (1 - c_0 \theta_1) z_2 - (\hat{\theta}_2 - \theta_2) x_1 + c_0(\hat{\theta}_1 - \theta_1) x_1 - c_0(\hat{\theta}_1 - \theta_1)\alpha_1 \quad (13)$$

The new Lyapunov function for (11) and (13) is:

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2}\gamma^{-1}(\hat{\theta}_1 - \theta_1)^2 + \frac{1}{2}\gamma^{-1}(\hat{\theta}_2 - \theta_2)^2$$

$$D^q V_2 \le -c_1 z_1^2 - \theta_1 z_1 z_2 + z_2 D^q z_2 + \gamma^{-1}(\hat{\theta}_1 - \theta_1)(D^q \hat{\theta}_1) + \gamma^{-1}(\hat{\theta}_2 - \theta_2)(D^q \hat{\theta}_2)$$
$$\le -c_1 z_1^2 - x_1 z_2 z_3 + z_2\{\hat{\theta}_1 z_1 - x_1 \alpha_2 + \hat{\theta}_2 x_1 - c_0 \hat{\theta}_1 x_1 - (1 - c_0 \hat{\theta}_1)\alpha_1 - (1 - c_0 \theta_1) z_2\} + (\hat{\theta}_1 - \theta_1)\gamma^{-1}[(D^q \hat{\theta}_1) + \gamma(c_0 x_1 - c_0 \alpha_1 - z_1) z_2] + (\hat{\theta}_2 - \theta_2)\gamma^{-1}\left((D^q \hat{\theta}_2) - \gamma x_1 z_2\right)$$

Here, $\hat{\theta}_1$ and $\hat{\theta}_2$ are the estimates of θ_1 and θ_2.γ is the adaptation gain. The virtual controllerα_2 can be selected as:

$$\alpha_2 = \hat{\theta}_1 + \hat{\theta}_2 - c_0 \hat{\theta}_1 + c_0(1 - c_0 \theta_1) \quad (14)$$

and the parameter update laws will be:

$$D^q \hat{\theta}_1 = -\gamma(c_0 x_1 - c_0 \alpha_1 - z_1) z_2,\ D^q \hat{\theta}_2 = \gamma x_1 z_2 \quad (15)$$

While choosing the expression for α_2, singularity problem due to state x_1 has been avoided by retaining the term $(1 - c_0 \theta_1) z_2$ and putting a constraint on c_0, such that $(1 - c_0 \theta_1) > 0$. The virtual controller in (14) and update laws in (15) leads to

$$D^q V_2 \le -c_1 z_1^2 - c_2 z_2^2 - x_1 z_2 z_3 \quad (16)$$

where$c_2 = (1 - c_0 \theta_1)$. Here, $\theta_1 > 0$, and hence one can have$-1 < c_0 \le 0 < 1/\theta_1$, such that, $c_1 = c_0 \theta_1 + \theta_1 > 0$ and $c_2 = (1 - c_0 \theta_1) > 0$. The derivative of z_3 can be expressed as,

$$D^q z_3 = u + x_1 x_2 - \theta_3 x_3 - D^q \alpha_2 \quad (17)$$

The Lyapunov function for (z_1, z_2, z_3) subsystem given in (11), (13) and (17), is selected as:

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2}\gamma^{-1}(\hat{\theta}_3 - \theta_3)^2 \quad (18)$$

and its derivative will be,

$$D^q V_3 \le -c_1 z_1^2 - c_2 z_2^2 - x_1 z_2 z_3 + z_3(u + x_1 x_2 - p_3 x_3 - D^q \alpha_2) + (\hat{\theta}_3 - \theta_3)\gamma^{-1}(D^q \hat{\theta}_3)$$

The final control law will be,

$$u = -c_3 z_3 + x_1 z_2 - x_1 x_2 + \hat{\theta}_3 x_3 + D^q \alpha_2 \quad (19)$$

which results in,

$$D^q V_3 \le -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - (\hat{\theta}_3 - \theta_3)\gamma^{-1}(D^q \hat{\theta}_3 + \gamma x_3 z_3)$$

The update law can be written as

$$D^q \theta_3 = -\gamma x_3 z_3 \quad (20)$$

which results into

$$D^q V_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 \quad (21)$$

From the above expressions, it can be concluded that the system (9) is asymptotically stable. The expression (21) ensures that the transformation variable for z_1, z_2 and $z_3$evolve to zero in restricted time which further leads to the regulation of states x_1, x_2 and x_3.

V. Simulation Results And discussion

The orders for FO differentiations are taken as $q_1 = q_2 = q_3 = q = 0.995$ and the design constants are:$c_0 = -0.1, c_3 = 5$ for first case and $c_0 = -0.2, c_3 = 30$ for the second case. The adaptation gainγ has been chosen as 2 and 8 in each case, respectively.The simulation time is $T_{sim} = 5\ s$ and time step $h = 0.005$. Figures2 and 3 depict the stabilization and parameter estimation. As evident from the displayed figures, the states x_1 and $x_2$evolve to zero in fixed time, and state x_3 is bounded. The parameter estimates remain bounded. With increase in value of γ, the adaptation of parameter estimates is accelerated, and is manifested by the simulation results.

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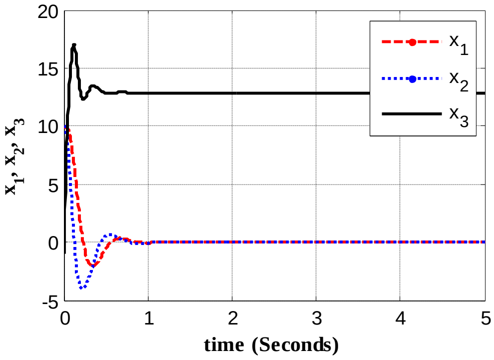
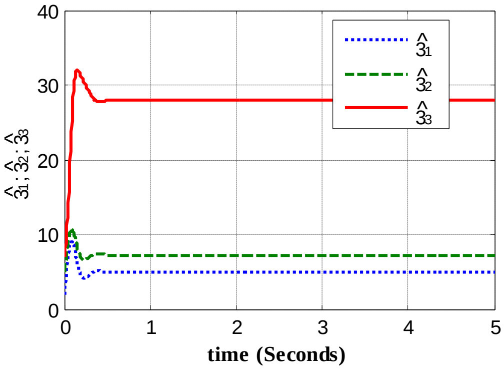
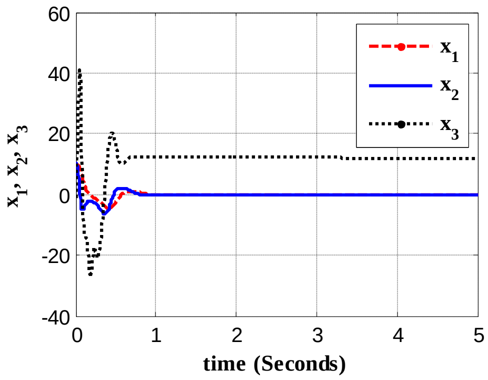


(a)

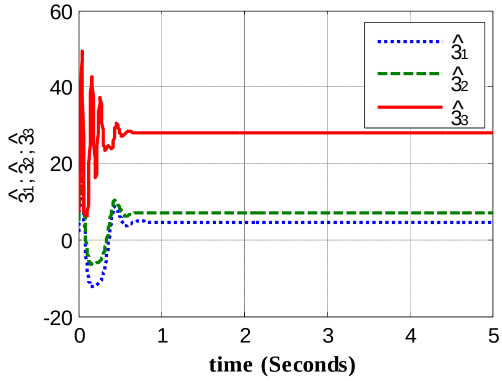


(b)

Fig. 2. (a) State stabilization and(b) parameter estimation for $\gamma = 2, c_0 = -0.1$ and $c_3 = 5$.



(a)



(b)

Fig. 3. (a) State stabilization and (b) parameter estimation for $\gamma = 8, c_0 = -0.2$ and $c_3 = 30$.

FO Lorenz system provides more realistic modelling of Lorenz system. In case of integer order modelling, one can only vary initial conditions and achieve different chaotic behaviors. But in case of FO modelling, one has a complete range of fractional order and hence with a slight variation in fractional order various chaotic behaviors can be obtained. Moreover, for different values of fractional order q, one can get different set of chaotic patterns which can provide additional security in various applications.

VI. Conclusion

The paper discusses the backstepping based step by step controller designer procedure for an uncertain FOCS which is in strict feedback form.On the basis of backstepping technique, a control technique for regulation of uncertain FO Lorenz system has been derived. Also, if this restriction is not forced then the states will diverge leading to the system instability. The proposed approach uses flexibility of backstepping method and avoids singularity behavior w.r.t the control action. The parameter update laws obtained while deriving the controller, give estimates of the uncertain parameters. The simulation results validate the proposed approach.

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