

Empirical Signatures For The Evolution of Collectivity Between Different Structural Symmetries In Nuclei

Amit Bindra

School of Chemical Engineering and Physical Sciences, Lovely Professional University,
Phagwara-144011, India

I present a modest method to discern the progression of nuclear structure from spherical to deformed in proton-proton, hole-hole and proton-hole region as a function of proton-neutron interaction strength. The present study emphasizes on the importance of p-n interaction strength which is responsible for configuration mixing and proposed a first direct correlation of empirical observables $\frac{1}{E(2_1^+)}$ and $B(E2: 2_1^+ \rightarrow 0_1^+)$ with $N_p N_n$ (valence nucleon product). The analogy is noticed about the growth rate of collectivity is higher in p-p, h-h region in contrast to p-h region. This approach is valuable for the future study of nuclei far from stability.

Keywords: atomic nuclei; nuclear structure; proton-neutron interaction; growth of collectivity.

1. Introduction

The development of collectivity in the nuclear structure with N and Z has a lengthy history. The p-n interaction has been main aspect in the evolution of deformation for over decades. First proposed in 1953 Goldhaber and De Shalit [1] improved the knowledge of nuclear structure by highlighting the dominance of p-n interaction and frequently emphasized by Talmi [2]. The scope of study was further extended by Federman-Pittel [3] on the $A = 100$ region where the sudden shape/phase transition in nuclear structure occurs, which was subsequently incorporated by Casten et al. [4] for $A = 150$ mass region with the elimination of $Z = 64$ subshell gap. The J. Dobaczewski et al. [5] further emphasized the orbit dependence of proton-neutron interaction in altering the shell structure. K. Heyde [6] and colleagues highlighted the importance of monopole component of p-n interaction and extended its applicability to shape phase coexistence by appearance of low lying intruder state. Casten et al. [7] discovered the first order shape phase transition (X5) symmetry which further emphasized the importance of p-n interaction. Finally, the study of valence proton-neutron interaction associated with the rise of collectivity and, ultimately deformation has been pointed out in $N_p N_n$ scheme [8,9]. To observe the additional rapid change in the structure the competition between p-n interaction and pairing (p-p and n-n) interaction is integrated in the P-factor [10]. Recently, Cakirli and Casten [11] pointed out the systematic dissimilarity in the growth of collectivity in various regions based on empirical measures that directly correlated to $R_{4/2}$. We caution here that $R_{4/2}$ independently does not influence the nuclear structure (except at extremes such as 2.0 for spherical shape and 3.33 for deformed rotor) which correspond in the terminology of interacting boson Model (IBM) [12] for the three dynamical symmetries $U(5)$, $O(6)$ and $SU(3)$ respectively. Stevenson et al. [13] has examined anomaly in $R_{4/2}$ at ^{190}W and studied the results with Hartree-Fock approach. Another observable of nuclear deformation is the ground state energy $E(2_1^+)$ which is mass dependent and decreases rapidly towards deformed regions of nuclei, but nevertheless act as a very valuable

indicator for deformation. It is always better to look for new systematic a important alternative [14] is $\frac{1}{E(2_1^+)}$ (which reflects in behavior of $R_{4/2}$) provide a information about the difference in growth rate of collectivity for the nuclear structure. This will also extend the scope of study for the exotic nuclei which are far from the stability. The present work advances the scope of existing knowledge about collectivity, deformation and quantum phase transition and give an idea about the step by step evolution of structure of even-even nuclei. The large focus will genuinely be on the regions of sharp structural change where proton-neutron interaction begins to predominate the pairing interaction and typify deformed shapes and closely have elegant link to Landau theory [15] and Ising-type models [16].

2. Theory

The study of nuclear structure for the whole $A \sim 100-200$ regions of medium mass nuclei are of great interest. The two important indicators of structure are to observe at the systematics of $\frac{1}{E(2_1^+)}$ and $B(E2)$ transition strengths. The $B(E2)$ values is the quadrupole ($E2$) transition probability between 2_1^+ and 0_1^+ states, $B(E2: 2_1^+ \rightarrow 0_1^+)$, which can be written in terms of $e^2 \text{fm}^2 = 10^{-2} e^2 b^2$ or in Weisskopf units (W.u.) act as an important indicator of the structure [17]. The systematics of nuclear shape/phase transitional region around $A \sim 100-200$ are examined from the view point of valance nucleon product $N_p N_n$. The experimental values of $E(2_1^+)$ are taken from National Nuclear Data Centre [18]. The reduced electric quadrupole transition probability $B(E2: 2_1^+ \rightarrow 0_1^+)$, values data taken from Pritychenko et al. [19].

3. Results and Discussion

We studied the variation of $\frac{1}{E(2_1^+)}$ and $B(E2: 2_1^+ \rightarrow 0_1^+)$ with $N_p N_n$ in different parts of subshell space for $Z = 50-82$ and $N = 82-126$. However, here we assume a clustering based on valance hole and particlepair consideration [20]. Thus, the $Z = 50-82$, $N = 82-126$ is divided into four subspaces: p-p, p-h, h-h and h-p. The last subspace is vacant. The present study leads to a significant understanding of these complex regions and pointed out that collectivity develop rapidly in proton-proton and hole-hole region as contrast to proton-hole region.

3.1 The $Z = 50-82$ and $N = 82-126$ entire region

In the study of the $A \sim 100-200$ for $\frac{1}{E(2_1^+)}$ vs $N_p N_n$ [See Fig.1] plotted over entire region $Z = 50-82$ and $N = 82-126$ reveals the parameter $\frac{1}{E(2_1^+)}$ increases empirically from nuclei near to magic number towards mid-shell (large $N_p N_n$ values). The three regions are compared i.e; p-p, p-h and h-h in $N_p N_n$ scheme. Anomaly is noticed about the dissimilarity in growth of collectivity depending upon p-n interaction strengths.

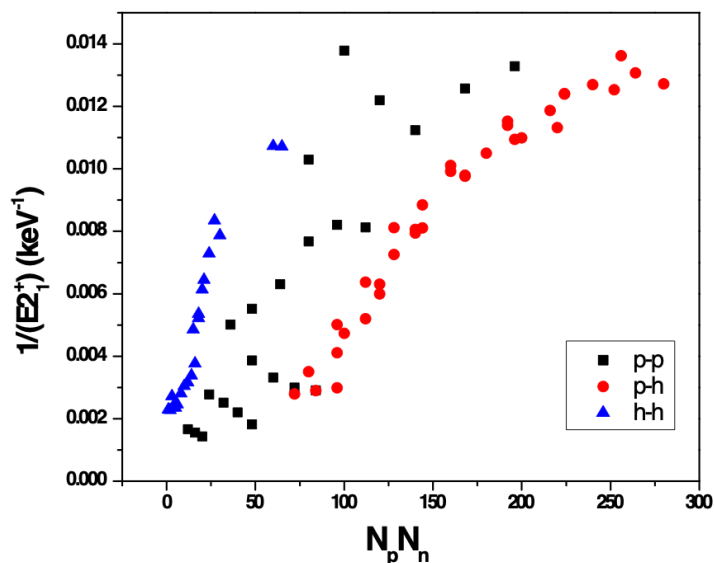


Figure 1: The systematics of $\frac{1}{E(2_1^+)}$ in units of keV^{-1} vs $N_p N_n$ for all nuclei p-p, h-h and p-h ($Z = 50-82$, $N = 82-126$)

The general trend was noticed in [Fig. 1(a) of Ref. 11] while analysing the plots of $R_{4/2}$ with $N_p N_n$. The anomaly is noticed when the factor $R_{4/2}$ is replaced by $\frac{1}{E(2_1^+)}$, the three regions (p-p, h-h and p-h) discern at different rate. However, the remarkable point in the present study is the discerning of curves that appear to be coalesced in previous reference. Note that there is insufficient $R_{4/2}$ data for three regions as compared to $\frac{1}{E(2_1^+)}$. The present approach highlights the hidden physics about the growth rate of collectivity is highest in h-h region and least in dissimilar nucleons p-h region. In fact, the benefit of the more available $E(2_1^+)$ data as compared to $R_{4/2}$ helps to gain more access in regions of exotic nuclei. It is also noticed that the $N_p N_n$ provides an exceptional measure to evaluate the immediacy of different transitional regions.

3.2 The comparison of $Z = 50-66$, $N = 82-104$ p-p subspace and $Z = 68-82$, $N=82-104$ p-h subspace.

In the study of the $Z = 50-82$, $N = 82-104$ region, (see Fig.2) the systematic of $\frac{1}{E(2_1^+)}$ plotted as a function of $N_p N_n$ reveals the intriguing phenomena that collectivity develop faster in p-p subspace compared to p-h subspace. The p-p subspace with $Z = 50-66$, $N = 82-104$ (Nd, Sm and Gd) belongs to transitional class of nuclei. For a fixed Z , as N changes from 88 to 90 and higher, the neutrons start filling the $h_{9/2}$ orbit and develop a proton polarization such that they partially occupy the $h_{11/2}$ orbital. The gain in neutron-proton interaction energy will be at peak and sudden rise in collectivity leads to deformed structure occurs at $N_p N_n \sim 50$. The sudden growth of collectivity among transitional nuclei is responsible for elimination of $Z = 64$ subshell gap [4].

The sudden onset of deformation in particle-particle subspace as also exhibited on the energy ratio $\frac{E(4_1^+)}{E(2_1^+)}$ versus $N_p N_n$ plots [4].

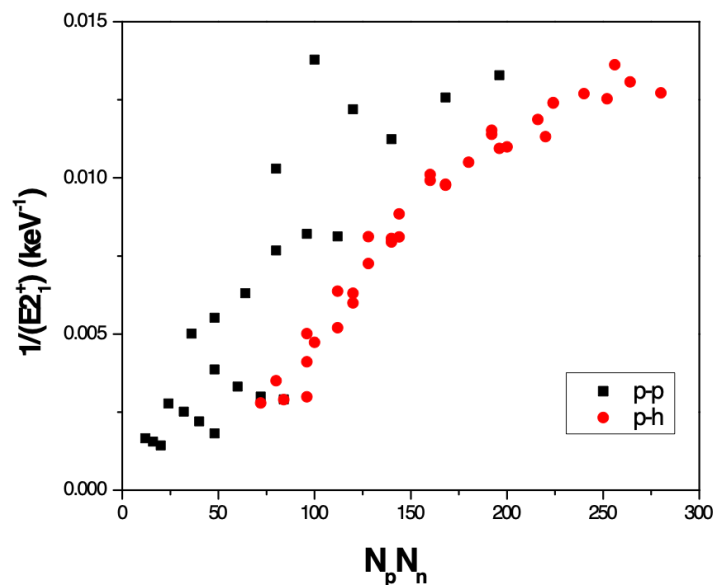


Fig. 2 : The systematics of $\frac{1}{E(2_1^+)}$ in units of keV^{-1} vs $N_p N_n$ for $Z = 50-66$, $N = 82-104$ p-p subspace and $Z = 68-82$, $N=82-104$ p-h subspace.

However, the $Z = 68-82$, $N=82-104$ p-h subspace (Dy, Er and Yb) generally belongs to well deformed region or SU(3) symmetry in language on IBA. The most striking feature of $Z>66$ is noticed for dissimilar portions (p-h) on $N_p N_n$ plot, the $h_{11/2}$ orbit is partially filled for all N values which reduces the scope of significant $Z = 64$ gap. The $Z>66$ appears to be well defined family and less deformed as compared to $Z<66$ on $N_p N_n$ plots. Gupta [20] also reported the similar behaviour in the context of Grodzins product $E(2_1^+) * B(E2) \uparrow$ constancy for ($Z>66$) well deformed family. However, reported the breakdown of Grodzins product $E(2_1^+) * B(E2) \uparrow$ constancy in the region of ($Z<66$) rapid deformation.

3.3 The comparison of $Z = 66-82$, $N = 82-104$ p-h subspace and $Z = 66-82$, $N = 106-126$ h-h subspace.

The systematic dependence of $\frac{1}{E(2_1^+)}$ vs $N_p N_n$ is studied in [Fig.3] for $Z = 66-82$ and $N = 82-126$ subspace. In this region as expected collectivity grows faster in like orbits (h-h) region as compared to dissimilar orbits (p-h) region. The $Z = 66-82$, $N = 106-126$ (h-h) subspace shows a striking correlation with very rapid growth in collectivity for $N_p N_n < 50$. The physical origin of rapid deformation in (Hf, W, Os and Pt) nuclei is easy to understand. There is well known subshell gap at $N = 114$, which can be easily obliterated as proton number exceeding 76, the

most recent protons enter the upsloping $\frac{11}{2}$ [505] and $\frac{7}{2}$ [402] orbits. Identical to the region $Z = 50-66$, $N = 82-104$ p-p subspace, the nuclei $N = 88$ are near to spherical, while those with $N > 90$ approach the deformed character.

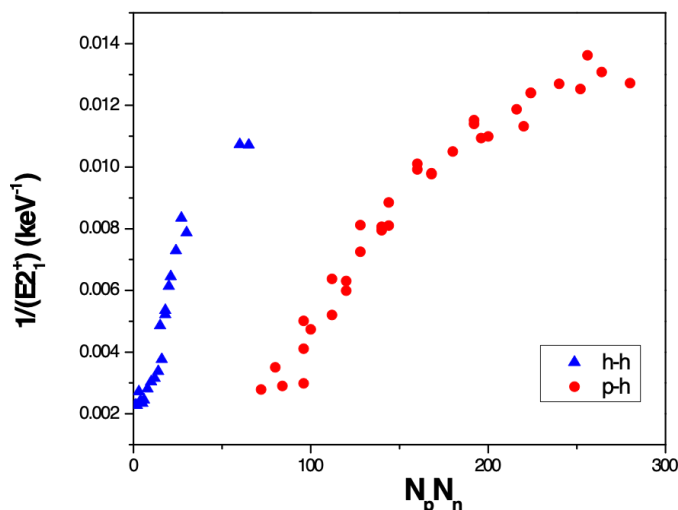


Fig.3 : The systematics of $\frac{1}{E(2_1^+)}$ in units of keV^{-1} vs $N_p N_n$ for $Z = 66-82$, $N = 82-104$ p-h subspace and $Z = 66-82$, $N = 106-126$ h-h subspace.

Here, the isotones below and above $N = 114$, are spherical for $Z > 76$ becomes deformed for $Z < 76$. Cakirli and Casten [11] stressed on the point that parity orbits in heavy nuclei filled from high j-low n to low j-high n nature. It clearly manifests that as a result of strong interactions similar orbits are filling fast.

3.4 $N_p N_n$ systematics of $B(E2)$ transition strengths

The second observable that behaves as an important signature of the structure are $B(E2)$ values. The data for $B(E2: 2_1^+ \rightarrow 0_1^+)$ transition plotted as function of $N_p N_n$ systematic shows alike pattern but with less scatter. The $B(E2: 2_1^+ \rightarrow 0_1^+)$ plot in all the three regions reflect the similar result as concluded with parameter $\frac{1}{E(2_1^+)}$ as a function of $N_p N_n$ that the rate of growth of collectivity is faster in similar nucleons (p-p and h-h) as compared to dissimilar nucleons (p-h). Cakirli and Casten [14] plotted $B(E2: 2_1^+ \rightarrow 0_1^+)$ values as a function of N , Z for the whole region $Z = 50-82$, $N = 82-126$ and reveals transition strengths are negligible near magic numbers and peaking near the mid-shell. Some other important $B(E2)$ values, as well, the important indicators of shape. In the earlier study of Gupta et al. [21] reveals a smooth and systematic dependence $\gamma \rightarrow g$ $B(E2)$ ratios in all the four regions on $N_p N_n$ product indicated important correlation to the shape/phase deformation.

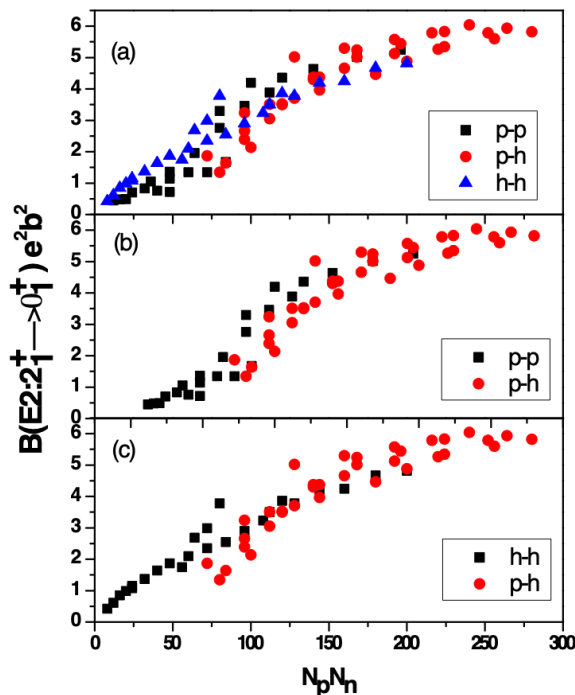


Fig. 4 $B(E2: 2_1^+ \rightarrow 0_1^+)$ calculations in $Z = 50-82$, $N = 82-126$ region plotted as a function of $N_p N_n$. (a) All subspaces (p-p, h-h and p-h); (b) correlation between p-p subspace ($Z = 50-66$, $N = 82-104$) and p-h subspace ($Z = 68-82$, $N = 82-104$); (c) correlation between h-h subspace ($Z = 68-82$, $N = 106-126$) and the p-h subspace.

4. Conclusion

We have highlighted the empirical evidence using $\frac{1}{E(2_1^+)}$ and $B(E2: 2_1^+ \rightarrow 0_1^+)$ experimental values plotted as a function of $N_p N_n$ (valence nucleon product) that identifies the rate of growth of collectivity is higher for (p-p or h-h) identical nucleons as compared to (p-h) dissimilar nucleons. The physical origin of rapid deformation is the neutron-proton interaction which is too large in similar nucleons (proton-proton and hole-hole) than dissimilar nucleons (proton-hole). Fig. 4 was very useful for analysing and complements p-h as a well-defined deformed family. The $B(E2)$ values were considered as a clear signature to identify spherical vibrational and rotational spectra nuclei. The present technique used $\frac{1}{E(2_1^+)}$ which help to discern the nuclear region with greater clarity as it was coalesced with $R_{4/2}$. The empirical parameters $\frac{1}{E(2_1^+)}$ and $B(E2: 2_1^+ \rightarrow 0_1^+)$ could be further utilize to study the exotic nuclei.

5. References

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