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Effect of Rotation and Analysis of Elasto-Diffusive Body Waves In Semiconductor Materials

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ABSTRACT

In the present paper specific loss factors, attenuations and phase velocities of three elasto-diffusive (EN) waves have been calculated. The irreducible case of Cardano's method has been implemented to solve complex coefficients cubic equation. The medium is considered to be homogenous isotropic, rotating, n-type semiconductor material with charge carrier fields. The numerical illustrations have been carried out for Germanium *Ge* semiconductor material and also represented graphically with help of MATLAB code.

KEY WORDS: Semiconductor, rotation, attenuation, Kibel number.

1. INTRODUCTION

Maruszewski [1-3] and Many et al. [4] have been studied, the interaction in semiconductors, the diffusion of charge carrier's fields in between thermo-elastic media and also have been formulated mathematically. Maruszewski [5] has formulated uncoupled cases to evaluate at very low temperatures, the velocities for thermal fields and the velocities in semiconductors, for diffusion at room temperatures dealing with charge carrier's fields.

Auriault [6] and Sharma and Grover [7] studied effect of rotation on the body wave in terms of ratio of wave frequency to angular frequency in elastic and thermoelastic media respectively. An analysis of waves in structure such as disks and beam, which are rotating also have been reported in literature [8-9].

In present paper, the analytical expressions of various wave characteristics namly. Specific loss, phase velocities and attenuation coefficients have been obtained for homogenous isotropic rotating, n-type semiconductor material with charge carrier fields, elastic media with help of Cardano's method.

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2. MATHEMATICAL FORMULATION OF PROBLEM

A rotating, homogenous isotropic semiconductor infinite elastic medium initially at undeformed state , with identical angular velocity $\vec{\Omega}$, which is rotating about $y-$axis, has been considered. In absence of body forces, the basic governing equations [5] after including the centripetal forces as well as Coriolis forces are

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\nabla\nabla.\vec{u}-\lambda^n\,\nabla N=\rho\left(\ddot{\vec{u}}+\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)+2\left(\vec{\Omega}\times\dot{\vec{u}}\right)\right) \tag{1}$$

$$\rho D^n\nabla^2N-\rho\left(1+t^n\frac{\partial}{\partial t}\right)\dot{N}-a_2^nT_0\lambda^T\nabla.\dot{\vec{u}}=-\left(1+t^n\frac{\partial}{\partial t}\right)\left(\frac{\rho}{t_n^+}\right)N \tag{2}$$

where the notations

$\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial z^2}$, $a_1^n=\dfrac{a_{Qn}}{a^Q}$, $a_2^n=\dfrac{a_{Qn}}{a^n}$, $N=n-n_0$, $T=T_1-T_0$, $\lambda^T=(3\lambda+2\mu)\alpha_T$ are used.

$\vec{u}(x,y,z,t)=(u_x,0,u_z)$	displacement vector
$\vec{\Omega}(x,\ z,\ t)=(0,\ \Omega,\ 0)$	angular frequency
$N(x,\ z,\ t)$	electron carrier field
λ,μ	Lame's parameter
λ^n	elasto-diffusive constant of electrons
$a^{Qn},\ a^n$	diffusion coefficients of electron
t^n	the relaxation time of electron fields
t_n^+	the life times of carrier field
ρ	density of semiconductor
n and n_0	the non-equilibrium , equilibrium values of electrons concentration respectively
α_T	linear thermal expansion coefficient
$T(x,\ z,\ t)$	temperature change
the field variables and parameters are also supposed to fulfill all constraints as pronounced by Maruszewski [5]	

Non-dimensional quantities are defined as below

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$$x' = \frac{\omega^* x}{c_1},\ z' = \frac{\omega^* z}{c_1},\ t' = \omega^* t,\ \ N' = \frac{N}{n_0},\ u' = \frac{\rho\omega^* c_1 u}{\lambda^T T_0},\ w' = \frac{\rho\omega^* c_1 w}{\lambda^T T_0},\ \ {t^n}' = \omega^* t^n,$$

$${t_n^+}' = \omega^* t_n^+,\ \Omega' = \frac{\Omega}{\omega^*},\ \delta^2 = \frac{c_2^2}{c_1^2},\ \omega^* = \frac{c_1^2}{D^n},\ c_1^2 = \frac{\lambda + 2\mu}{\rho},\ c_2^2 = \frac{\mu}{\rho},$$

$$\in_n = \frac{a_2^n T_0 \lambda^T \lambda^n}{\rho(\lambda + 2\mu)} \tag{3}$$

Introducing the quantities (3) in equations (1)-(2), the results are as follows:

$$\delta^2 \nabla^2 \vec{u} + (1-\delta^2)\nabla\nabla.\vec{u} - \nabla N = \ddot{\vec{u}} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right) + 2\left(\vec{\Omega}\times\dot{\vec{u}}\right) \tag{4}$$

$$\nabla^2 N + \left\{\frac{1}{t_n^+} N - \left(1 - \frac{t^n}{t_n^+}\right)\dot{N} - t^n \ddot{N}\right\} - \in_n \nabla.\dot{\vec{u}} = 0 \tag{5}$$

To proceed with simplicity, primes have been inhibited.

3. SOLUTION OF DISPERSION EQUATION

Considering the direction of rotation along $y-$axis i.e $\vec{\Omega} = \Omega\hat{j}$. Therefore, $\vec{u}$ is collinear to $\vec{\Omega}$ Hence, analysis of displacements carried out in $xz-$plane, which will remain fixed along $y-$axis. Considering $\phi = \nabla.\vec{u}$ and $\vec{\psi} = \nabla\times\vec{u} = \psi_2\hat{j}$ and applying consequently on equation (4) which yields the following differential equations which are coupled in terms of ϕ and ψ_2 as

$$\left[\nabla^2 + \Omega^2 - \frac{\partial^2}{\partial t^2}\right]\phi + 2\Omega\dot{\psi}_2 - \nabla^2 N = 0 \tag{6}$$

$$\left(\delta^2\nabla^2 + \Omega^2 - \frac{\partial^2}{\partial t^2}\right)\psi_2 - 2\Omega\dot{\phi} = 0 \tag{7}$$

The coupled system of equations includes equations (6)-(7) along with equation (5).
Considering the waves propagating in $x-$direction as normal mode form given below

$$(\phi, \psi_2,\ N) = (\bar{\phi}, \bar{\psi}_2,\ \bar{N})e^{\iota(kx-\omega t)},\quad \iota^2 = -1 \tag{8}$$

Using normal mode solution (8) in equations (5) to (7), we get

$$\left[1 - v^2\left(1+\Gamma^{-2}\right)\right]\bar{\phi} + 2\iota v^2\Gamma^{-1}\bar{\psi}_2 - \bar{N} = 0 \tag{9}$$

$$2\iota v^2\Gamma^{-1}\bar{\phi} - \left[\delta^2 - v^2\left(1+\Gamma^{-2}\right)\right]\bar{\psi}_2 = 0 \tag{10}$$

$$-\iota\omega^{-1} \in_n v^2\bar{\phi} + \left(1 - \tau_n^* v^2\right)\bar{N} = 0 \tag{11}$$

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where $\tau_n^* = \left[\frac{\omega^{-2}}{t_n^+} + \left(1 - \frac{t^n}{t_n^+}\right) \iota\omega^{-1} + t^n \right]$, $v = \frac{\omega}{k}$. Here $\Gamma = \frac{\omega}{\Omega}$ is defined as Kibel number.

The non-trivial solution system of equations (9)-(10) for $\bar{\phi}, \bar{\psi}_2$ and $\bar{N}$ in view of existing condition yields the following dispersion equation

$$\prod_{i=1}^{3} (1 - v^2 \xi_i^2) = 0 \tag{12}$$

Here ξ_i^2, $i = 1, 2, 3$ represents the zeros of the cubic equation

$$\xi^6 - A\xi^4 + B\xi^2 - C = 0 \tag{13}$$

where

$$A = \frac{\left(1 + \Gamma^{-2}\right)\left(1 + \delta^2\right)}{\delta^2} + \tau_n^* + \iota\omega^{-1} \in_n$$

$$B = \left(1 + \Gamma^{-2}\right)\left(1 + \frac{1}{\delta^2}\right)\tau_n^* + \iota\omega^{-1} \in_n \left(\frac{1 + \Gamma^{-2}}{\delta^2}\right) + \frac{\left(1 - \Gamma^{-2}\right)^2}{\delta^2}$$

$$C = \frac{\left(1 - \Gamma^{-2}\right)^2}{\delta^2} \tau_n^* \tag{14}$$

The Cardano's method has been used with help of DeMovire's Theorem to solve equation (13) being cubic equation in ς^2 outlined as follows:

Using the transformation

$$\Lambda^2 = \varsigma^2 - \frac{A}{3} \tag{15}$$

to remove the second term, so that the coefficient of highest degree term is unity in transformed equation, the equation (13) becomes

$$\Lambda^6 + 3H\Lambda^2 + G = 0 \tag{16}$$

where $H = \frac{3B - A^2}{9}$

$$G = \frac{9AB - 27C - 2A^3}{27}$$

Let

$$\Lambda^2 = \upsilon + \varpi \tag{17}$$

Cubing both sides of equation (17), we have

$$\Lambda^6 - 3\upsilon\varpi\,\Lambda^2 - \left(\upsilon^3 + \varpi^3\right) = 0 \tag{18}$$

Comparing coefficients of this equation with those of equation (16), we get

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$$\upsilon^3+\varpi^3=-G\,,\ \upsilon^3\varpi^3=-H^3 \tag{19}$$

Therefore, υ^3 and ϖ^3 are roots of the quadratic equation

$$\Xi^2+G\Xi-H^3=0 \tag{20}$$

which are given by

$$\upsilon^3=\frac{-G+\sqrt{G^2+4H^3}}{2}$$

$$\varpi^3=\frac{-G-\sqrt{G^2+4H^3}}{2} \tag{21}$$

Because G and H are complex quantities, so we shall use DeMovire's theorem of trigonometry to evaluate the expressions (21). Obviously,

$$\upsilon^3=\kappa_1+\iota\kappa_2=\hat{\kappa}(\cos\varphi+\iota\sin\varphi) \tag{22}$$

where

$$\hat{\kappa}=\sqrt{\kappa_1^2+\kappa_2^2}$$

$$\varphi=\tan^{-1}\left(\frac{\kappa_2}{\kappa_1}\right)$$

Applying De Moivre theorem, we get

$$\upsilon=\hat{\kappa}^{\frac{1}{3}}\left[\cos\left(\frac{\varphi+2k\pi}{3}\right)+\iota\sin\left(\frac{\varphi+2k\pi}{3}\right)\right],k=0,\ 1,\ 2 \tag{23}$$

Also

$$\upsilon\varpi=-H$$

Upon using roots (23) in this equation and simplifying, we get

$$\varpi=-\frac{H}{\upsilon}=-|H|\hat{\kappa}^{\frac{1}{3}}\left[\cos\left(\frac{\varphi+2k\pi}{3}-\tilde{\varphi}\right)-\iota\sin\left(\frac{\varphi+2k\pi}{3}-\tilde{\varphi}\right)\right],k=0,\ 1,\ 2 \tag{24}$$

where $|H|=\sqrt{H_R^2+H_I^2}\,,\ \tilde{\varphi}=\tan^{-1}\left(\frac{H_I}{H_R}\right), H_R=\mathrm{Re}(H),\ H_I=\mathrm{Im}(H)$

Substituting for υ and ϖ in equation (17) and then using in equation (15), we get

$$\varsigma_k^2=\frac{A}{3}+\hat{\kappa}^{\frac{1}{3}}\left[\begin{array}{l}\left\{\cos\left(\frac{\varphi+2k\pi}{3}\right)+\iota\sin\left(\frac{\varphi+2k\pi}{3}\right)\right\}\\ -|H|\left\{\cos\left(\frac{\varphi+2k\pi}{3}-\tilde{\varphi}\right)-\iota\sin\left(\frac{\varphi+2k\pi}{3}-\tilde{\varphi}\right)\right\}\end{array}\right],\ k=0,\ 1,\ 2 \tag{25}$$

For $\Gamma=1, C=0$, the equation (13) reduces to

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$$\hat{\varsigma}^4 - A^*\hat{\varsigma}^2 + B^* = 0,\ \hat{\varsigma}^2 = 0 \tag{26}$$

Where

$$A^* = 2\left(1+\frac{1}{\delta^2}\right) + \tau_n^* + \iota\omega^{-1} \in_n$$

$$B^* = \left(\frac{2}{\delta^2}\right)\left(\left(1+\delta^2\right)\tau_n^* + \iota\omega^{-1} \in_n\right)$$

Upon obtaining the complex roots ς_i^2 , $i = 1,\ 2,\ 3$ of equation (13) and (26), we get

$$v_j = \begin{cases} \pm\dfrac{1}{\varsigma_j}, & \Gamma \neq 1 \\ \pm\dfrac{1}{\hat{\varsigma}_j}, & \Gamma = 1 \end{cases} \qquad (j = 1,\ 2,\ 3) \tag{27}$$

However, we may take

$$\hat{\varsigma}_1 \to 0$$

$$\hat{\varsigma}_2 = \left[\frac{A^* + \sqrt{A^{*2} - 4B^*}}{2}\right]^{1/2}$$

$$\hat{\varsigma}_3 = \left[\frac{A^* - \sqrt{A^{*2} - 4B^*}}{2}\right]^{1/2}$$

in the following analysis.

If we write

$$v_j^{-1} = V_j^{-1} + \iota\omega^{-1}Q_j,\ j = 1,\ 2,\ 3 \tag{28}$$

Hence both V_j and Q_j are real.

The following expressions have been obtained by including representation (28) in equation (27)

$$V_j = \begin{cases} \dfrac{1}{\mathrm{Re}\left(\varsigma_j\right)} \\ \dfrac{1}{\mathrm{Re}\left(\hat{\varsigma}_j\right)} \end{cases} \qquad (j = 1,\ 2,\ 3)$$

$$Q_j = \begin{cases} \omega\,\mathrm{Im}\left(\varsigma_j\right) \\ \omega\,\mathrm{Im}\left(\hat{\varsigma}_j\right) \end{cases} \qquad (j = 1,\ 2,\ 3) \tag{29}$$

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For non-zero angular frequency the amplitude ratios for existing waves given

$$\left(\frac{\overline{\psi}_2}{\overline{\phi}}\right)_i = \frac{2\iota\,\Gamma^{-1}v_i^2}{\delta^2 - v_i^2\left(1+\Gamma^{-2}\right)},\quad i = 1,\,2,\,3$$

$$\left(\frac{\overline{N}}{\overline{\phi}}\right)_i = \frac{\iota\omega^{-1}\in_n v_i^2}{\left(1-\tau_n^* v_i^2\right)} = \frac{\left[\delta^2 - v_i^2\left(1+\Gamma^{-2}\right)\left(1+\delta^2\right)+v_i^4\left(1-\Gamma^{-2}\right)^2\right]}{\left(\delta^2 - v_i^2\left(1+\Gamma^{-2}\right)\right)},\ i = 1,\,2,\,3 \tag{30}$$

4. SPECIFIC LOSS

According to Kolsky [10], the mathematical expression for specific loss can be written as

$$SL_i = \left|\frac{\Delta E}{E}\right| = 4\pi\left|\frac{\mathrm{Im}(k)}{\mathrm{Re}(k)}\right| = 4\pi\left|\frac{V_i Q_i}{\omega}\right|,\ i = 1,\,2,\,3 \tag{31}$$

5. NUMERICAL ILLUSTRATIONS AND GRAPHICAL PRESENTATION

The graphical presentations have been completed in view of illustrating analytical results and some numerical computations and simulations have been performed for Germanium (*Ge*), the semiconductor material with the availability of physical data (Maruszewski [5]). Under the assumptions that the semiconductor is of the relaxation type so that the diffusion approximation of physical process ceases to be obligatory and $t_n,\, t_n^+$ turn comparable to each other in their values.

From Figure 1, it has been observed that in Germanium semiconductor material, the velocity of quasi-longitudinal (QL) wave attains resonance for equal value of wave and angular frequencies. Initially value increases to attain maximum value followed by decrease in value to attain stable value for $\Gamma \geq 3$. The behaviour of QT-wave, which will increase as ratio of wave frequency and angular frequency increases and finally approaches to stable value δ for its large values. The velocity of EN wave detects growing trend for increasing values of the ratio of wave frequency to angular frequency. The magnitude of quasi-longitudinal wave is of the order of (10^4).

From Figure 2, It has been observed that for Germanium (*Ge*) semiconductor material, the profile of quasi-longitudinal wave in terms of attenuation coefficient rises for the ratio of wave frequency to angular frequency in range $0 < \Gamma < 1$ until it becomes balanced and unwavering for $\Gamma > 1$. The profile of quasi-transverse wave in terms of attenuation perceives falling trend initially for $0 < \Gamma < 0.5$ followed by sharp rise in the ratio of wave

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frequency to angular frequency in range $0.5 < \Gamma < 1$ and afterwards trend becomes asymptotic nature and approaches balanced and unwavering for large values of the ratio of wave frequency to angular frequency. The EN wave observes the larger attenuation in comparison to QL and QT waves. It is also observed that the EN wave attains fixed value of 4.96*10^3 for all values of Kibel number. The QT and EN waves having the magnitude of the order of (10^-1) and (10^3) respectively for profile in terms of attenuation coefficient

Figure 3 reveals that for Germanium (*Ge*) semiconductor material, the specific loss factor of EN and quasi-longitudinal waves rises with the developing values of ratio of wave frequency to angular frequency and it is observed that the magnitude of EN wave is greater than that of quasi-longitudinal wave. The specific loss factor of quasi-transverse wave growths in the range $0 \le \Gamma \le 0.5$ to reach its maximum value at $\Gamma = 0.5$ and falls for $0.5 < \Gamma < 3$ and tends to the stable value for $\Gamma \ge 3$. The magnitude of specific loss factors of quasi-transverse wave is of the order of (10^-3).

CONCLUSIONS

It is observed that the life time of charge carriers significantly affect the characteristics of elasto-diffusive waves in Germanium (*Ge*) semiconductor material. In semiconductor materials, at the room temperature the treatment of diffusion fields can be valuable in many applications. The rotation and electron fields have significant affect in the media under analysis. The different wave characteristics viz. phase velocity, attenuation and specific loss factors have been investigated and analysis shows that these characteristics have substantial variations in comparison to the different characteristics of elasto-diffusive (EN) waves in rotating media.

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Figure 1: Variations of Phase velocity versus Kibel number

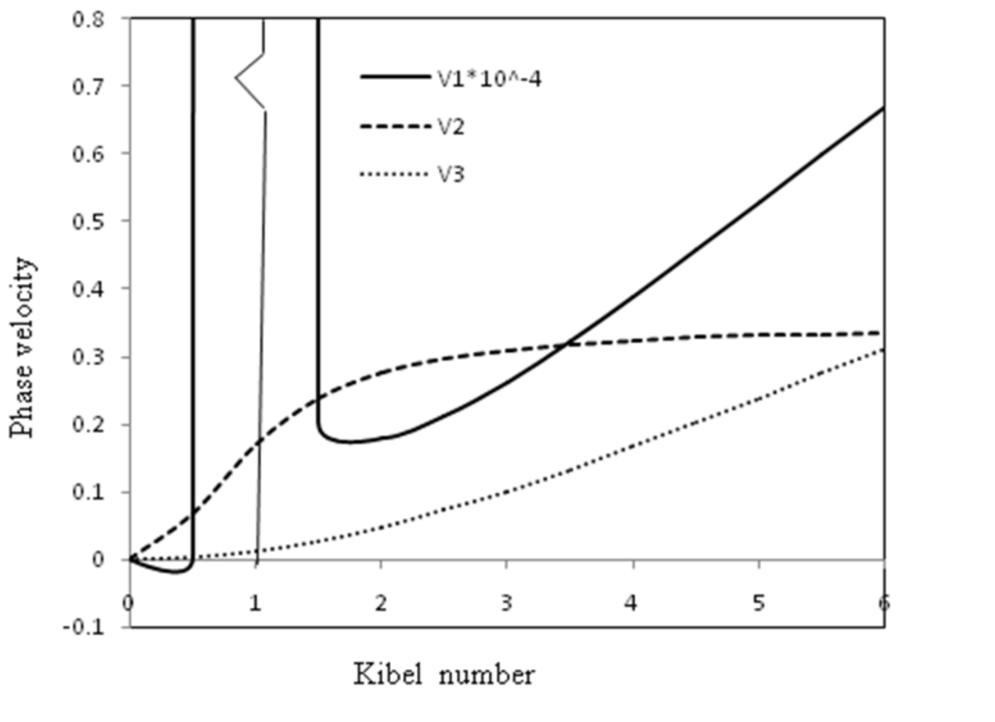

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Figure 2: Variations of Attenuation versus Kibel number

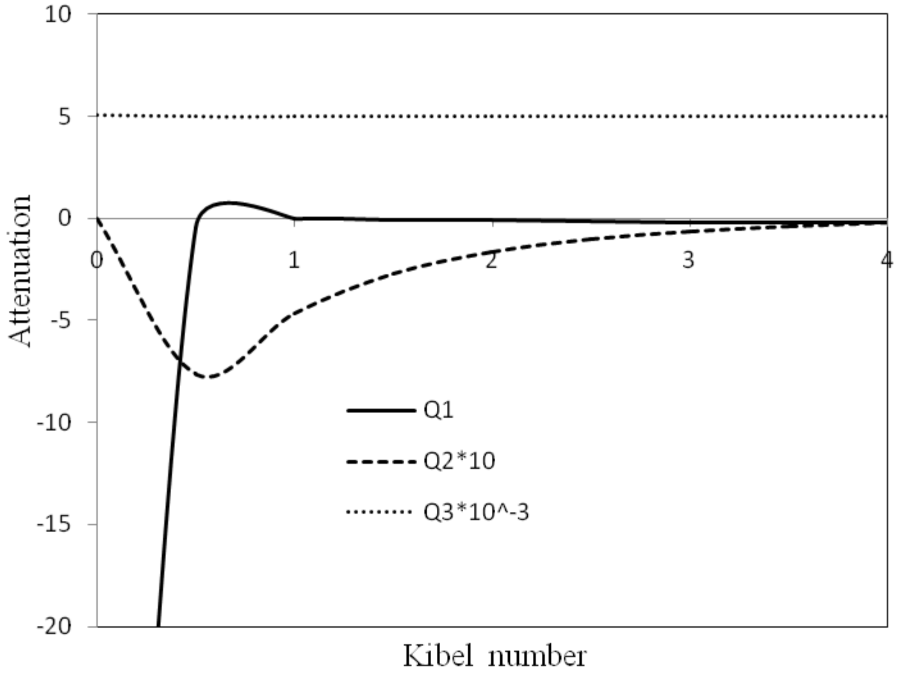

Figure 3: Variations of Specific loss versus Kibel number