

Comparison of Torsion of Non-Circular Shafts of Different Cross-Sections Using Finite Element Analysis

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Abstract

A shaft is a structural member used to transmit torque. For most of the application, especially those involving high torque, shafts with circular cross-section are used as stress distribution is uniform along the radius in all the directions. However, in stationary torque applications, due to ease in power transmission and assembly shafts of non-circular sections are also used. This paper compares the torsion of a rectangular, circular, triangular and elliptical shaft using the finite element tool Ansys Workbench.

1. Introduction

Non-circular shafts find their presence in stationary torque applications[1]. For transmitting power in machines and automobiles, circular shafts are used. However, there are many applications where non-circular members experience due to direct or indirect loading. Shafts used in motorized robotic arms, 3d printers are usually having square cross-section as the square cross-section makes the assembly simple. Chassis of automobiles is also made up of rectangular members and during turning, they also experience torsional moments. Aircrafts and other mobile components, while turning, are also subjected to torsional moments on their bodies. So, analysis of stress and deformation in non-circular members subjected to torque becomes an important aspect while designing them.

The torsion theory assumes that plane sections remain plain after deformation and this assumption holds true only for circular section. For members non-circular sections, the section deforms out of plane and the type of deformation is commonly referred to as warping. Figure 1 (a) and (b) shows the undeformed and deformed view of the rectangular view of the shaft when subjected to torque about the axis passing through the centroid.

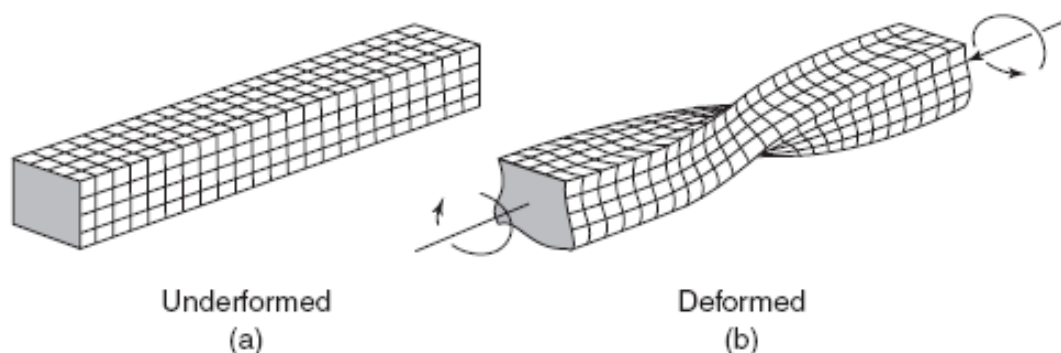


Figure 1 Deformation of a rectangular shaft[1]

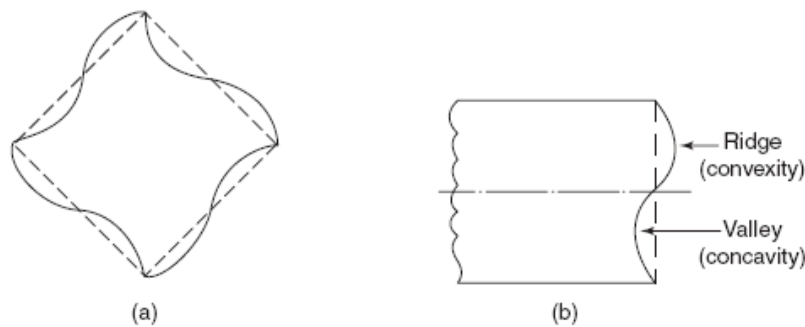


Figure 2 Changes in cross-section[1]

Figure 2 (a) and (b) shows different orthographic views of the section. Figure 2 (a) shows the deformation in plane of section and 2 (b) shows the deformation in a plane perpendicular to cross-section. As represented in Figure 2 (b) some point move out from the cross-sectional plane and move in the opposite direction. This type of deformation is called as warping.

This paper provides the deformation and stresses in shafts of non-circular sections with circular sectioned shaft using popular FE package Ansys Workbench[2]. For analysis, cross-sectional area of the shaft has been kept constant.

Following dimensions have been used:

Table 1 Dimensions

Cross-section	Width/radius (mm)	Height (mm)	Area
Circle	11.28379167		100
Square	10	10	100
Rectangle	8	12.5	100
Equilateral Triangle	15.19693661		100
Triangle	20	10	100

For all the sections, centroid has been taken at origin for analysis.

2. Related Work

Design of shaft is based on torsion theory. Torsion of non-circular sections can be determined analytically using St Venant's Semi-Inverse method[3]. The approximate solution of torsion problem of homogenous elastic members has been studies by many researchers[4]. The solution of irregular section using Fredholm integral equation on an artificial circle, which just encloses the bar's cross-section has been proposed by Chein-Shan Liu[5]. The author has used a meshless regularized integral equation method to attain the solution of the problem. Semi-analytical method along with conjugate gradient method has been used by author and using few Fourier terms calculations have been completed. Hematiyan and Doostfatemeh[6] have presented an approach for torsional analysis of hollow non-circular sections based on Prandtl's stress function.

3. Finite Element Modeling

Popular FEA software Ansys Workbench Academic 2019R3 [2] has been used to carry out finite element modeling and determine stresses and deformation. The modeling has been done in Ansys

Design Modeler where a box of specified size is made. The length of shaft has been taken along z-axis.

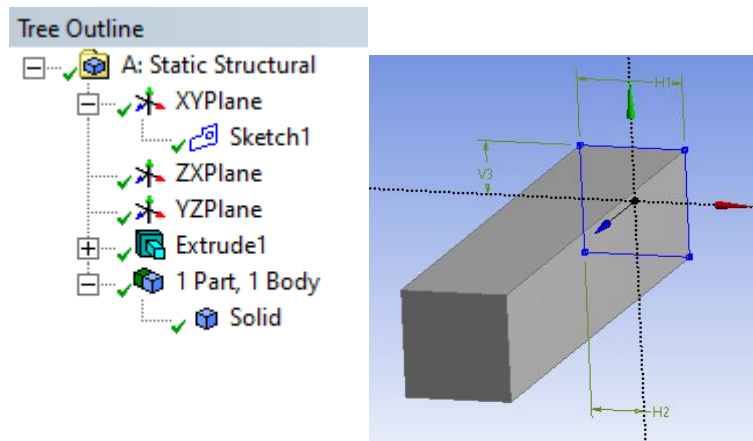


Figure 3 Modeling of Shaft

After modeling, meshing was performed using Automatic mesh with quadratic elements of 1 mm element size. One end of the shaft was fixed using fixed support option and other end was subjected to a moment of 20 kN-m about z-axis. The option of components was chosen for applying the moment.

Displacement along z-axis and shear stress result was requested during calculation. For obtaining the angular rotation, APDL commands were inserted[7]. A remote point was inserted in the model and its position was extracted using following APDL command

Measure_Pilot=_npilot

Finally, the rotation of the point (in degrees) was obtained by adding following command in solution.

*my_xrotation=ROTX(Measure_Pilot)*57.29577951*

*my_yrotation=ROTY(Measure_Pilot)*57.29577951*

*my_zrotation=ROTZ(Measure_Pilot)*57.29577951*

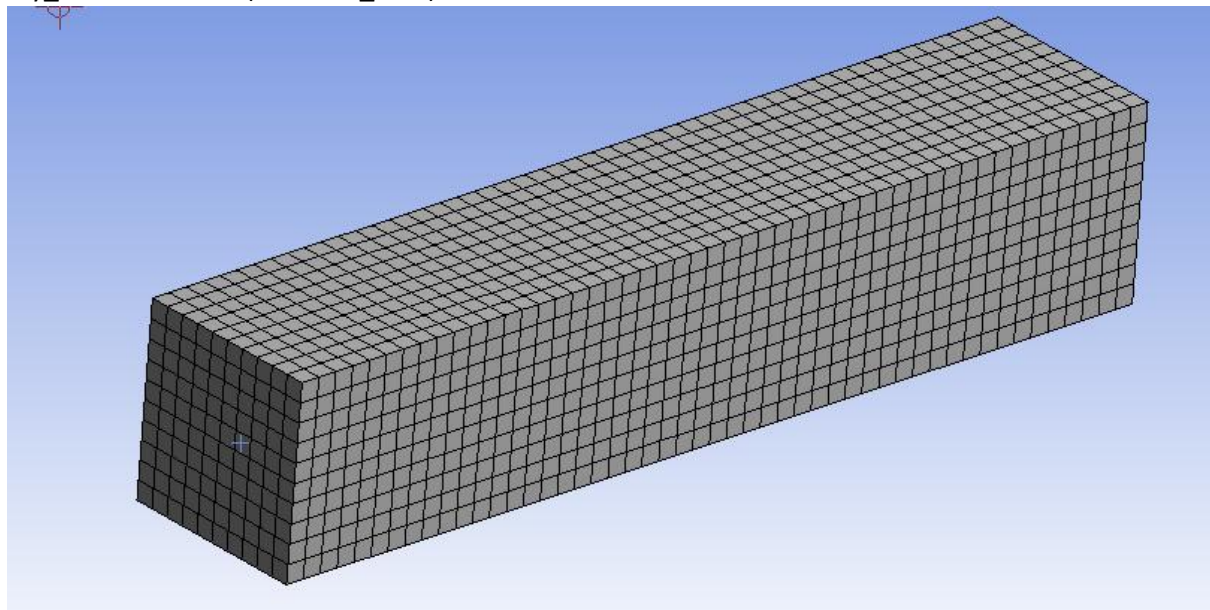


Figure 4 Discretized Geometry

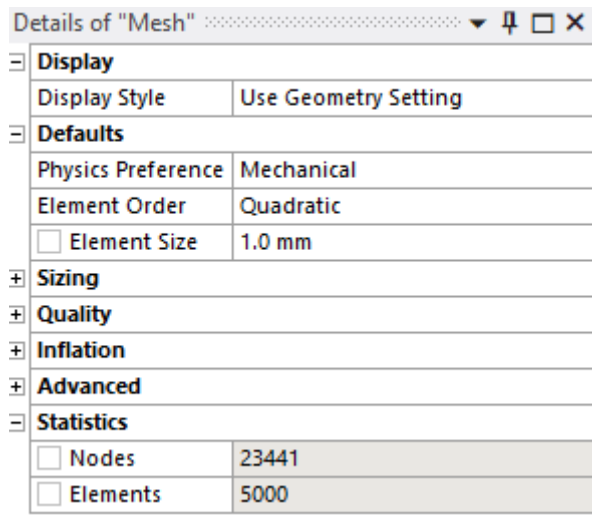


Figure 5 Mesh Properties and Statistics

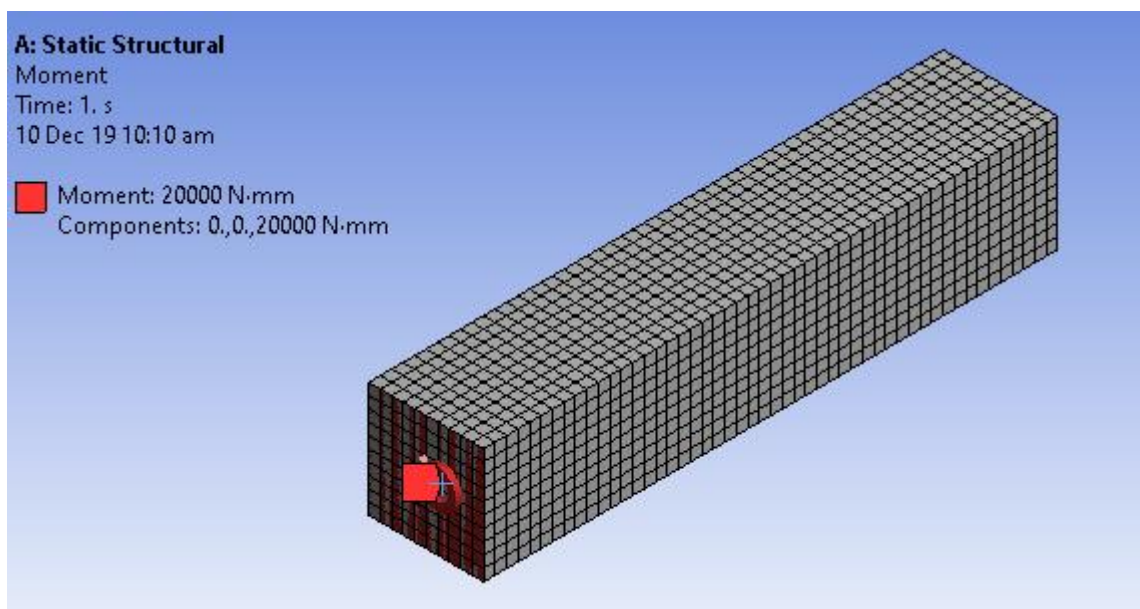


Figure 6 Loading of shaft

4. Results and Discussion

The results as obtained from Ansys Workbench are listed in table 2. As observed, shaft with circular cross-section has minimum displacement and stresses. The distribution of all the parameters is also uniform. Further, when cross-section is symmetrical, distribution of stress and displacement is uniform, and magnitude is also less than similar cross-section with non-uniform section.

Table 2 FEM Results

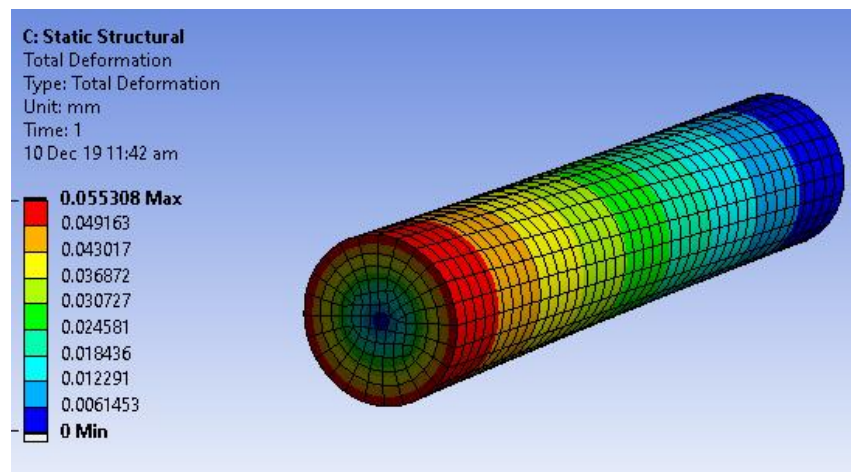
Cross-section	Max displacement in z-direction (mm)	Max resultant displacement (mm)	Von Mises Stress (MPa)	Twist (degrees)
Circle	0.000000314	0.055	123.82	0.52

Square	0.001	0.066	182	0.533
Rectangle	0.003	0.091	205.14	0.69
Equilateral Triangle	0.002	0.122	258.52	0.722
Triangle	0.0042	0.1706	313.45	0.875

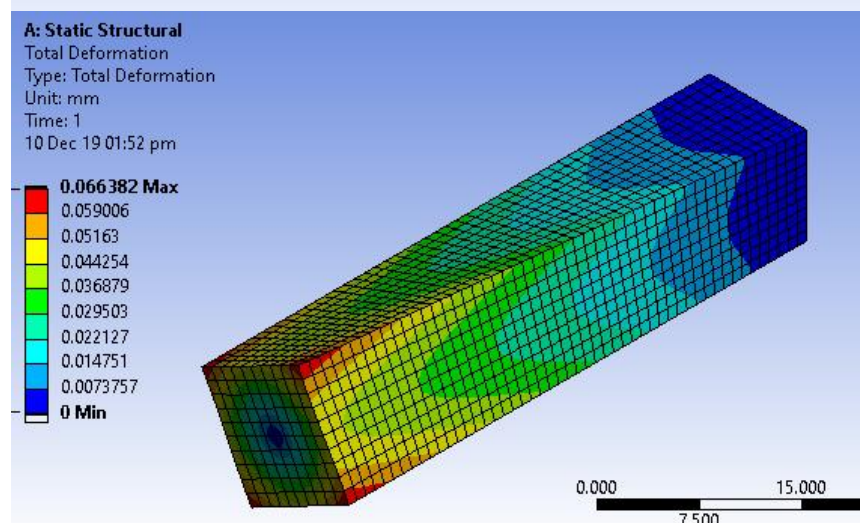
Cross-section

Total Deformation

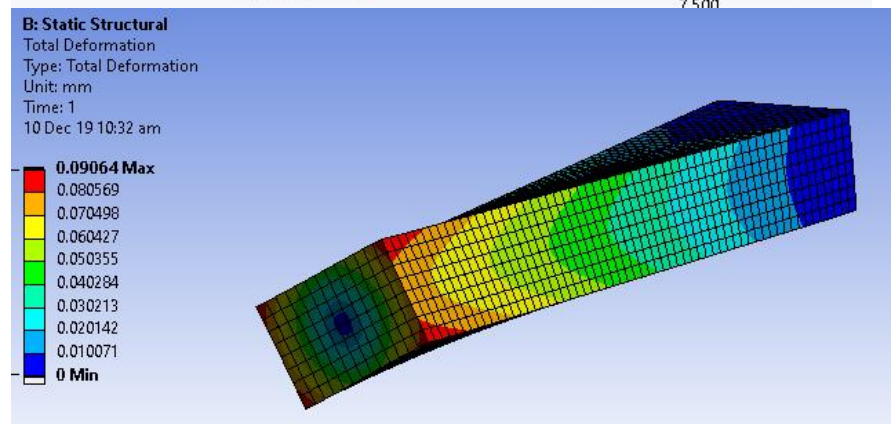
Circle



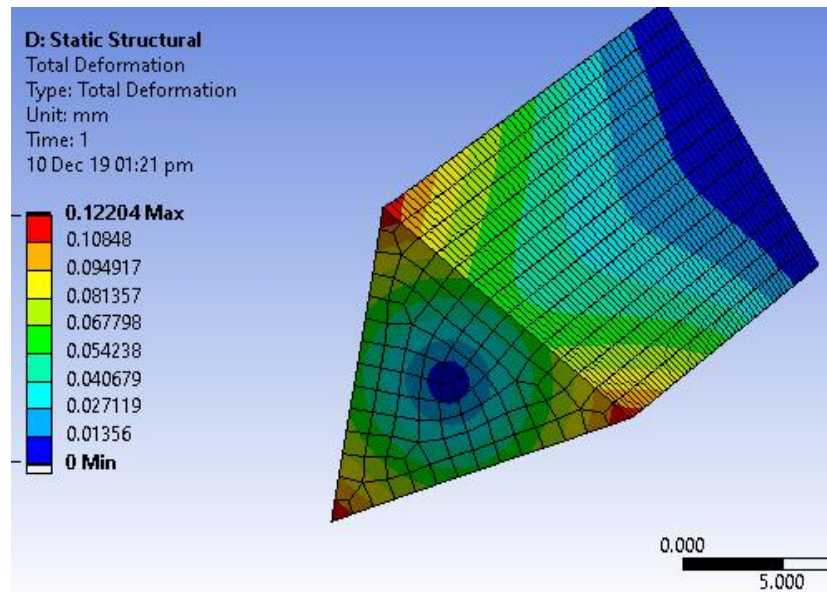
Square



Rectangle



Equilateral
Triangle



Triangle

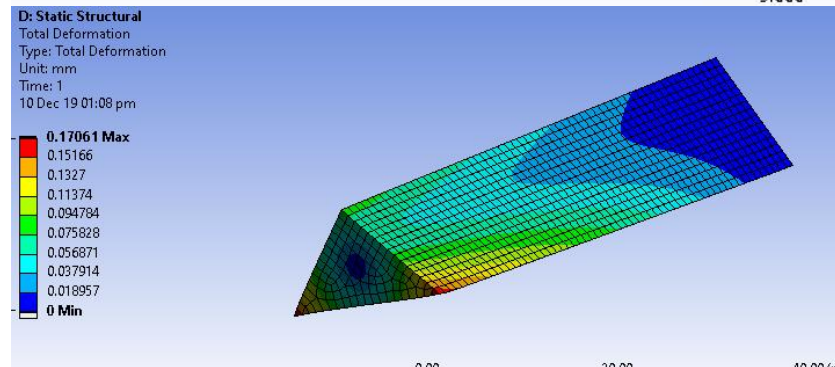
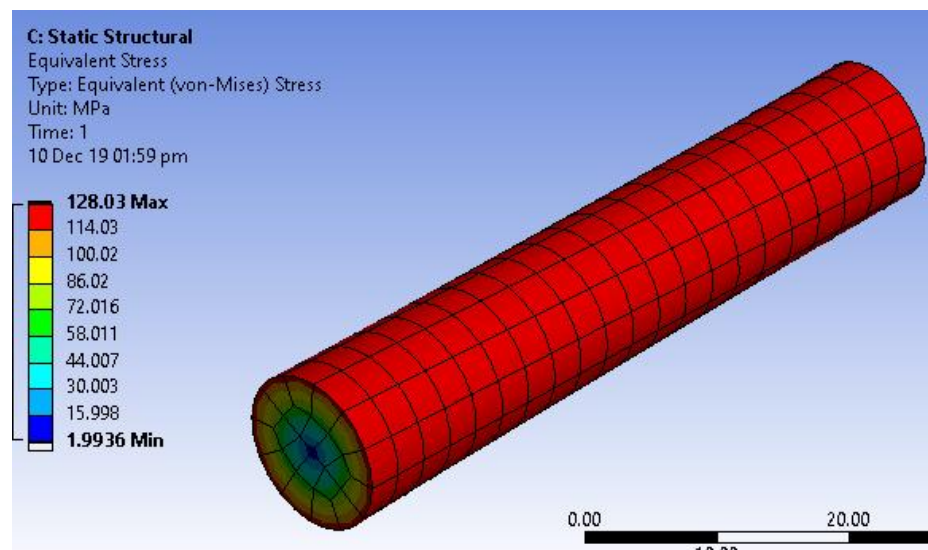


Figure 7 Comparison of Total Deformation

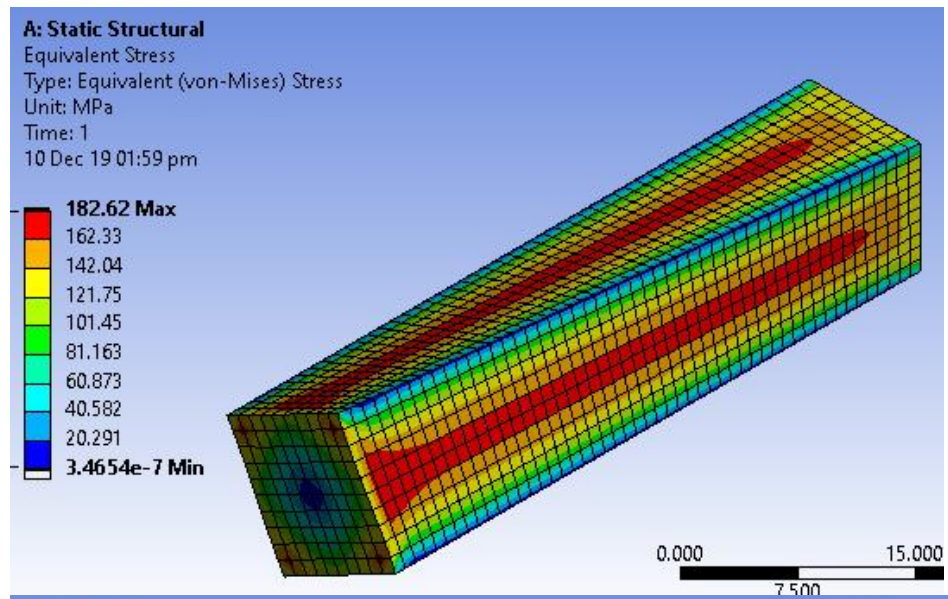
Cross-section

Maximum Von-Mises Stress

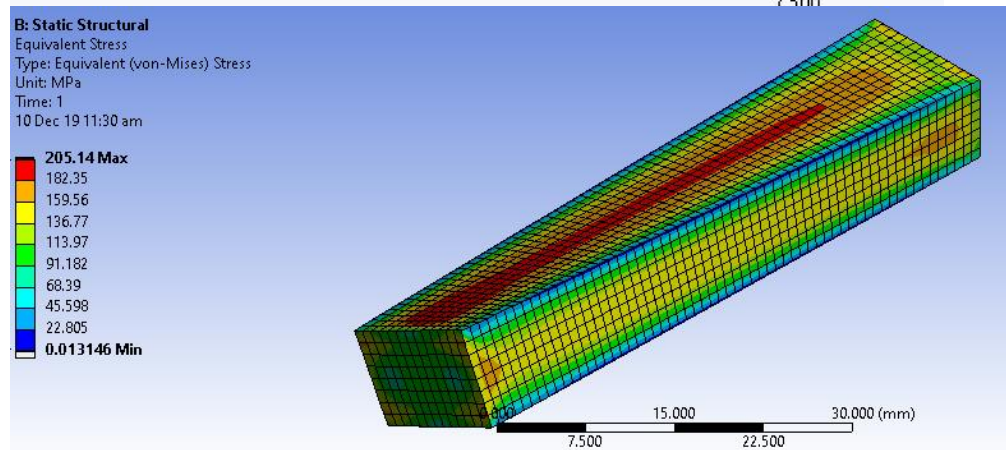
Circle



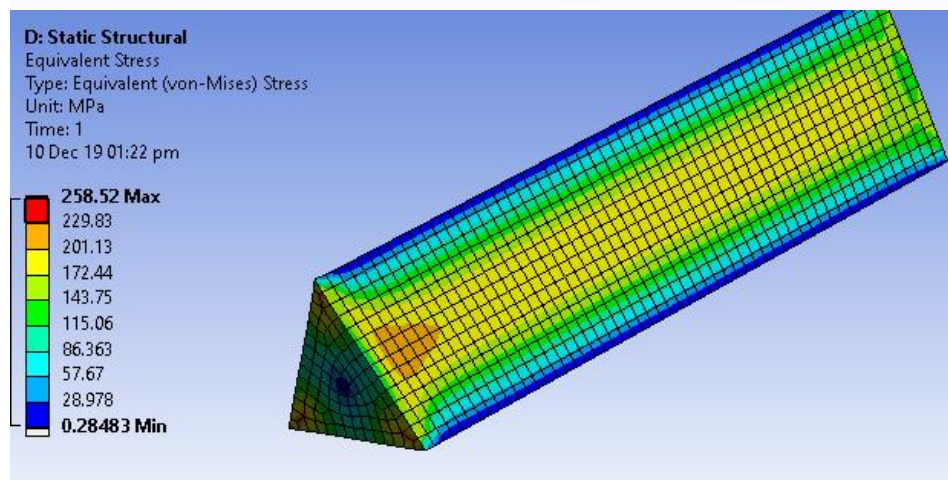
Square



Rectangle



Equilateral Triangle



Triangle

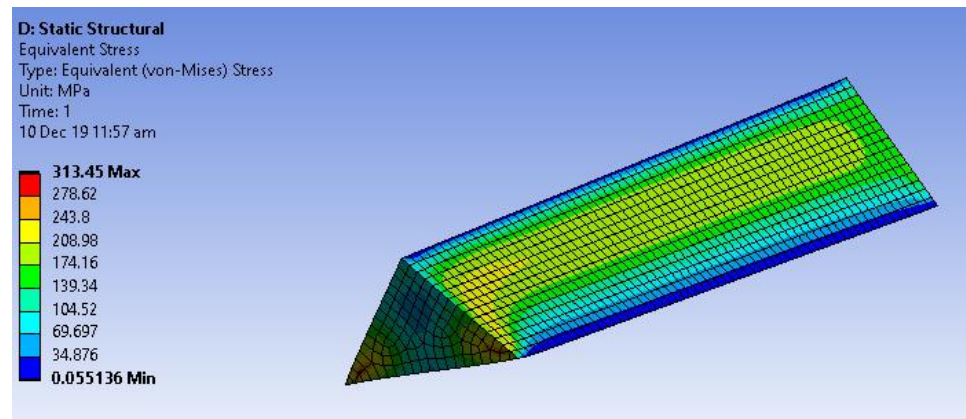


Figure 8 Comparison of Von-Mises Stress in different cross-section

$$\sigma_{\text{square}} = 88.7\% \text{ of } \sigma_{\text{rectangle}}$$

$$\sigma_{\text{equilateral triangle}} = 82.5\% \text{ of } \sigma_{\text{triangle}}$$

From the comparative study, it is concluded that where-ever possible shafts used should be of circular cross-section. If non-circular shafts are to be used, they should have symmetrical cross-section to have uniform stress and displacement distribution.

References

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