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Thestudy of Fuzzy Differential Equation With Triangular, Trapezoidal and Quadrilateral Initial Conditions Using Fuzzy Laplace Transforms.

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Abstract

In this paper, different formulations of the fuzzy differential equation are solved using the fuzzy Laplace transform. The radioactive decaying differential equation underimposed fuzzy initial conditions is considered and three different systems of fuzzy differential equations for right angled triangular, trapezoidal and quadrilateral initial conditions are prepared and solved using fuzzy Laplace transform. A new quadrilateral fuzzy number is introduced and using that the decaying differential equation is explored. We established the fuzzy Laplace transform for all defined right angled triangular, trapezoidal and quadrilateral fuzzy numbers. The lower central and upper solutions according the applied fuzzy initial conditions are presented. The solution in terms of membership grade has also illustrated. The core effect of time on number of *radionuclide in* the sampleand α^*value in $\alpha^* - left$ quadrilateral fuzzy number$(Q_{c,i_c,d_1,d_2})$is investigated.

Introduction

The simulation of physical and engineering problems such as dynamic processes in solid and fluid dynamics, viscoelastics andanatomy leads to the Mathematical modelling through differential equations. The parameters and main conditions of a model are generally considered to be described accurately. In fact owing to calculation, analysis or investigation errors, the data on factors and conditions can be ambiguous or inelegant for some realistic modelling. In order to counter uncertainty or imprecision, a floated system can be utilized by transforming general differential equations into fuzzy differential equations.Several analyses were pursued for the last

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few decades on the various types of fuzzyderivatives and differential equations. The derivatives of fuzzy functions were examined in [1] thena theorem that provides a solution to a fuzzy differential equation was established [2].The general characteristics of the Sumudu transformation and the Sumudu transformation of the first order equation were discussed in [3]. Abbasbandy and Viranloo [4] analysed the Taylor scheme numerical computations for solving “fuzzy ordinary differential equations.New set of results for fuzzy differential equations are presented in [5] using stacking theorem .Regan*et al*.[6] did the nonlinear analysis of fuzzy differential equitation with initial and boundary conditions.

The generalized Hukuhara difference and fuzzy interval partition was numerically explored to approximate the fuzzy initial value problem [7].While a few more research to solve the fuzzy system of differential equation have been carried out in [9], [10] and [12-14].Basic characteristics of Sumudhu and Laplace transform were described in [11] and [15] respectively.Allahviranloo and Salahshourpresented Eulers technique and new approaches like fuzzy Laplace to solve the fuzzy hybrid,integro and fractional differential equations in [16-19]. F.V.I. method was employed to obtain the approximate results of nonlinear fuzzy differential equation [20], [25].Mondal and Roy [21] considered the coefficient and initial condition of differential equation as triangular fuzzy number and fuzzy Laplace method is implemented to solve the first order equation and then fuzzy nth order derivatives were evaluated in [22].SeveralIntuitionist differential equations solutions were carried out in [23].Nayak and Chakraverty [24] numerically approximated the fuzzy stochastic differential equation.Some solutions of resolving a complex and linear fuzzy differential equation were derived by You and Zhang [26] and then stability analysis of credibility for FDEs was also done in [28]. Gholami*et al*.[27] provided the approach to solve the 2-point boundary-value problem by fuzzy kernel method.

2. Preliminaries

2.1: Fuzzy Set and its Components

Definition 2.1.1: LetX^*be the universal spaceand a fuzzy set$\tilde{A}$, is a set in which each element of the set$X^*$is associated with a membership grade defined as:

$\tilde{A}$ = {(x^*, $\mu_{\tilde{A}}(x^*)$): $x^* \epsilon X^*$, $\mu_{\tilde{A}}(x^*)$) → 0 1]}

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Definition2.1.2: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]for\alpha^* \in \mu_{\tilde{A}}(x^*), then \alpha^* - cut$ of$\tilde{A}$is defined as $\tilde{A}^{\alpha^*} = \{x^* | \mu_{\tilde{A}}(x^*) \geq \alpha^*, x^* \epsilon X^*\}$

Definition 2.1.3: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ for$\alpha^* \in \mu_{\tilde{A}}(x^*), then strong \alpha^* - cut$ of$\tilde{A}$ is defined as $\tilde{A}^{\alpha^{*+}} = \{x^* | \mu_{\tilde{A}}(x^*) > \alpha^*, x^* \epsilon X^* \}$

Definition 2.1.4: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ for$\alpha^* \in \mu_{\tilde{A}}(x^*)$, then the support of $\tilde{A}$ is defined as

$\tilde{A}^s = \{x^* | \mu_{\tilde{A}}(x^*) > 0, x^* \epsilon X^* \}$

Definition2.1.5: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ for$\alpha^* \in \mu_{\tilde{A}}(x^*)$, then the height of $\tilde{A}$ is defined as $H(\tilde{A}) = max(\mu_{\tilde{A}}(x_i^*))$ and $\tilde{A}$ is said to be normal if $H(\tilde{A}) = 1$.

Definition 2.1.6: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ then it is called convex, if for any two $x_i^*, x_j^* \epsilon X^*$

$\mu_{\tilde{A}}\{ \lambda x_i^* + (1-\lambda) x_j^* \} \geq \min\{\mu_{\tilde{A}}(x_i^*), \mu_{\tilde{A}}(x_j^*)\}$,Where $0 \leq \lambda \leq 1$.

Definition 2.1.7: Let a set of fuzzy number$\varphi^* = \{\varphi : \Re \to [0, 1]$ if

1) φ is normal
2) φ is convex $\forall x_i^*, x_j^* \epsilon X^*$
3) upper semi-continuous on$\Re \in X^*$
4) $\tilde{A}^s$ and its closure are compact

Definition 2.1.8: The r-level of the fuzzy number $\varphi is defined as$

$\varphi^r = \{x^* | \mu_{\tilde{A}}(x^*) \geq r\ 0, x^* \epsilon X^* \}$

Definition 2.1.9: Let $\varphi_1$and $\varphi_2 \in \varphi^*$, $\exists \varphi_3 \in \varphi^*$ such that $\varphi_1 = \varphi_2 + \varphi_3$ then φ_3 is known as of $Hukuhara, H - difference$ of φ_1 and φ_2 defined by$\varphi_1 \ominus \varphi_2$ its r-level if

$$(\varphi_1 \ominus \varphi_2)^r = [\underline{\varphi_1}^r - \underline{\varphi_2}^r,\ \overline{\varphi_1}^r - \overline{\varphi_2}^r]$$

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3. Methodology

Definition 3.1: Fuzzy differentiation: A mapping $:\psi(\alpha_1,\alpha_2) \to \varphi$ is strongly generalized differentiable at $x_0 \in (\alpha_1,\alpha_2)$ if $\exists$an element $\omega^{'}(x_0) \in \varphi^*$ such that

$(i) For a small k > 0\ \exists \omega(x_0+k) \ominus \omega(x_0), \omega(x_0) \ominus \omega(x_0-k)$

$$and \lim_{k\to 0^+} \frac{\omega(x_0+k) \ominus \omega(x_0)}{k} = \lim_{k\to 0^+} \frac{\omega(x_0) \ominus \omega(x_0-k)}{k} = \omega^{'}(x_0)$$

$(ii) For a small k > 0\ \exists \omega(x_0-k) \ominus \omega(x_0), \omega(x_0) \ominus \omega(x_0+k)$

$$and \lim_{k\to 0^+} \frac{\omega(x_0) \ominus \omega(x_0+k)}{-k} = \lim_{k\to 0^+} \frac{\omega(x_0) \ominus \omega(x_0+k)}{-k} = \omega^{'}(x_0)$$

$(iii) For a small k > 0\ \exists \omega(x_0+k) \ominus \omega(x_0), \omega(x_0-k) \ominus \omega(x_0)$

$$and \lim_{k\to 0^+} \frac{\omega(x_0+k) \ominus \omega(x_0)}{k} = \lim_{k\to 0^+} \frac{\omega(x_0-k) \ominus \omega(x_0)}{-k} = \omega^{'}(x_0)$$

$(iv) For a small k > 0\ \exists \omega(x_0) \ominus \omega(x_0+k), \omega(x_0) \ominus \omega(x_0-k)$

$$and \lim_{k\to 0^+} \frac{\omega(x_0) \ominus \omega(x_0+k)}{-k} = \lim_{k\to 0^+} \frac{\omega(x_0) \ominus \omega(x_0-k)}{k} = \omega^{'}(x_0)$$

Definition 3.2: Fuzzy Laplace Transform: Let *ω (x)* is a continuous fuzzy membership grade function and$e^{-\beta x}\omega\,(x)$is improper fuzzy Riemann integrable on [0 ∞) then the fuzzy Laplace transform of$\omega(x)$ $is\ defined\ as \widetilde{\mathcal{L}}\,(\omega\,(x)) = \int_0^\infty e^{-\beta x}\omega\,(x)dx\,\beta > 0$

Theorem 3.3: Let $\omega'\,(x)$ is an integrablefuzzy membership grade functionon [0 ∞) then

(a) if $\omega\,(x)$ is (i)-differentiable:$\widetilde{\mathcal{L}}\,[\,\omega'(x)]$=$\beta * \omega(x) \ominus \omega(0)$
(b) if $\omega\,(x)$ is (ii)-differentiable: $\widetilde{\mathcal{L}}\,[\,\omega'(x)]$=$\big(-\omega(0)\big) \ominus (-\beta * \omega(x))$

3.4: Triangular fuzzy number: τ_{b,d_c,i_c}

Let *X* be the universal space of real number and $a\,, b\ and\ \ c\ \epsilon\ X\ such\ that\ a < b < c$ then a triangular fuzzy number$\tau_{b,d,i}$with membership grade $\tau_{b,d,i}(x)$ is defined as

$$\tau_{b,d_c,i_c}(x)=\begin{Bmatrix} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ \frac{c-x}{c-b} & b < x < c \\ 0 & x > c \end{Bmatrix} \text{ where} d_c = b - a, i_c = c - b.$$

And graph 1 represents the membership grade of $\tau_{b,d,i}$

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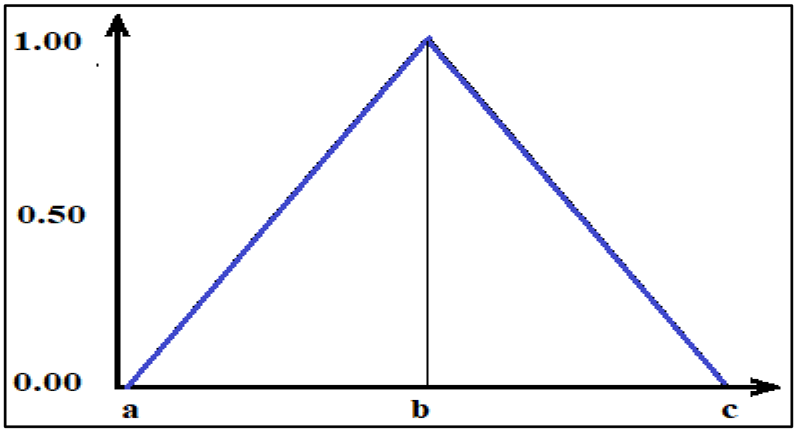

Graph1: Triangular fuzzy number τ_{b,d_c,i_c} with its membership grade

And the fuzzy Laplace transformation of $\tau_{b,d,i}$ is defined as

$$\widetilde{\mathcal{L}}\left(\tau_{b,d_c,i_c}\right) = \left(\widetilde{\mathcal{L}}\,\underline{\left(\tau_{b,d_c,i_c}\right)}, \widetilde{\mathcal{L}}\,\overline{\left(\tau_{b,d_c,i_c}\right)}\right)$$

$$\widetilde{\mathcal{L}}\left(\tau_{b,d,i}\right) = \left\{\begin{matrix} 0 & x < a \\ \widetilde{\mathcal{L}}\,\underline{\left(\tau_{\tau_{b,d_c,i_c}}\right)} & a < x < b \\ \widetilde{\mathcal{L}}\,\overline{\left(\tau_{b,d_c,i_c}\right)} & b < x < c \\ 0 & x > c \end{matrix}\right\} \text{ Where}$$

$$\widetilde{\mathcal{L}}\,\underline{\left(\tau_{b,d_c,i_c}\right)} = \frac{e^{-a\beta} - \left((b-a)\beta+1\right)e^{-b\beta}}{\beta^2(b-a)}$$

$$\widetilde{\mathcal{L}}\,\overline{\left(\tau_{b,d_c,i_c}\right)} = \frac{e^{-c\beta} - \left((c-b)\beta-1\right)e^{-b\beta}}{\beta^2(c-b)}$$

3.5: Triangular fuzzy number: $\tau_{b,i}$

Let X be the universal space of real number and $a\,, b \text{ and } c\,\epsilon\, X \text{ such that } a < b < c$ then a triangular fuzzy number $\tau_{b,i}$ with membership grade $\tau_{b,i}(x)$ is defined as

$$\tau_{b,i}(x) = \left\{\begin{matrix} 1 & x < a \\ \frac{b-x}{b-a} & a \le x \le b \\ 0 & x > b \end{matrix}\right\} \text{where } i = b - a.$$

And graph 2 represents the membership grade of $(\tau_{b,i})$

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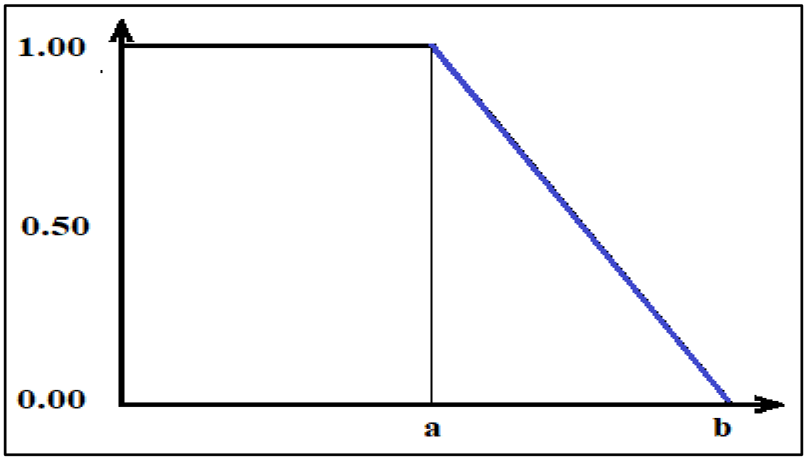

Graph2: Triangular fuzzy number $\tau_{b,i}$ with its membership grade

And the fuzzy Laplace transformation of $\tau_{b,i}$ is defined as $\widetilde{\mathcal{L}}\left[\tau_{b,d,i}\right] = \left[\widetilde{\mathcal{L}}\left(\tau_{b,i}\right)^{*}, \widetilde{\mathcal{L}}\overline{\left(\tau_{b,i}\right)}\right]$

$$\widetilde{\mathcal{L}}\left(\tau_{b,d,i}\right)=\begin{Bmatrix} \widetilde{\mathcal{L}}\left(\tau_{b,i}\right)^{*} & a < x \\ \widetilde{\mathcal{L}}\overline{\left(\tau_{b,i}\right)} & a < x < b \\ 0 & x > b \end{Bmatrix} \text{ Where}$$

$$\widetilde{\mathcal{L}}\left(\tau_{b,i}\right)^{*}=\int_0^a e^{-\beta x}.1dx=\frac{1-e^{-a\beta}}{\beta}, \beta > 0 \text{ similarly } \widetilde{\mathcal{L}}\overline{\left(\tau_{b,i}\right)} = \frac{e^{-b\beta}-\left((b-a)\beta-1\right)e^{-a\beta}}{\beta^2(b-a)}$$

3.6: Trapezoidal fuzzy number: $\tau_{b\sim c,d_c,i_c}$

Let X be the universal space of real number and $a, b, c \text{ and } d \in X \text{ such that } a < b < c < d$

Then a trapezoidal fuzzy number $\tau_{b\sim c,d,i}$ with membership grade $\tau_{b\sim c,d_c,i_c}(x)$ is defined as

$$\tau_{b\sim c,d_c,i_c}(x)=\begin{Bmatrix} 0 & x < a \& x > d \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b < x < c \\ \frac{d-x}{d-c} & c \le x \le d \end{Bmatrix} \text{where } d_c = b-a, i_c = d-c$$

And graph 3 represents the membership grade of $(\tau_{b\sim c,d_c,i_c})$

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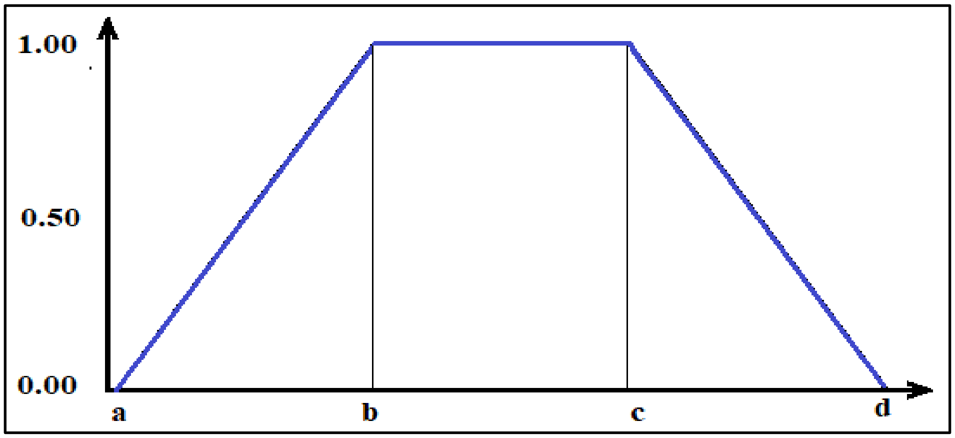

Graph3: Trapezoidal fuzzy number$\tau_{b\sim c,d_c,i_c}$ with its membership grade

And the fuzzy Laplace transformation of $\tau_{b\sim c,d,i}$ is defined as

$$\widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)=\left(\underline{\widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)},\widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)^*,\widetilde{\mathcal{L}}\overline{\left(\tau_{b\sim c,d_c,i_c}\right)}\right)$$

$$\widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)=\begin{Bmatrix} 0 & x<a\&x>d \\ \underline{\widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)} & a\le x<b \\ \widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)^* & b\le x<c \\ \widetilde{\mathcal{L}}\overline{\left(\tau_{b\sim c,d_c,i_c}\right)} & c<x\le d \end{Bmatrix} \text{Where}$$

$$\widetilde{\mathcal{L}}\underline{\left(\tau_{b\sim c,d_c,i_c}\right)}=\frac{e^{-a\beta}-\left((b-a)\beta+1\right)e^{-b\beta}}{\beta^2(b-a)}$$

$$\widetilde{\mathcal{L}}\left(\tau_{b\sim c,d_c,i_c}\right)^*=\frac{e^{-b\beta}-e^{-a\beta}}{\beta}$$

$$\widetilde{\mathcal{L}}\overline{\left(\tau_{b\sim c,d_c,i_c}\right)}=\frac{e^{-d\beta}-\left((d-c)\beta-1\right)e^{-c\beta}}{\beta^2(d-c)}$$

3.7: $\alpha-left$Quadrilateral fuzzy number:$Q_{c,i_c,d_1,\ d_2}$

Let X be the universal space of real number and $a\,,b,c\ and\ \ d\ \epsilon\ X\ such\ that\ a<b<c<d$

Then α^*-left Quadrilateral fuzzy number:Q_{c,i_c,d_1,d_2}With membership grade$Q_{c,i_c,d_1,\ d_2}(x)$ is defined as

$$Q_{c,i_c,d_1,\ d_2}(x)=\begin{Bmatrix} 0 & x<a\&x>d \\ \left(\frac{x-a}{b-a}\right)*\alpha^* & a\le x\le b \\ \alpha^*+\left(\frac{x-b}{c-b}\right)\overline{\alpha^*} & b<x<c \\ \frac{d-x}{d-c} & c\le x\le d \end{Bmatrix}$$

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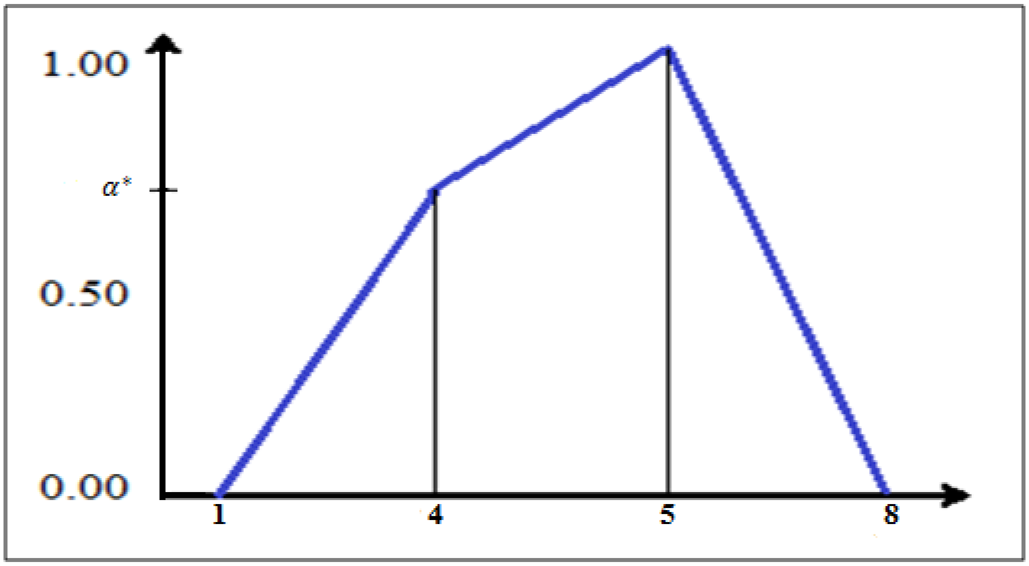
Where$i_c = d - c\,, d_1 = b - a,\ d_2 = c - b, \alpha^* \in [0.75\ 1], \overline{\alpha^*} = 1 - \alpha$.And graph 3 represents the membership grade of $(\boldsymbol{Q}_{\boldsymbol{c},i_c,d_1,\ d_2})$



Graph4: $\alpha^* - left$ Quadrilateral fuzzy number:$Q_{c,i_c,d_1,d_2}$with its membership grade

And the fuzzy Laplace transformation of Q_{c,i_c,d_1,d_2} is defined as

$$\widetilde{\mathcal{L}}\left(Q_{c,i_c,d_1,d_2}\right)=\left(\widetilde{\mathcal{L}}\,\underline{\left(Q_{c,i_c,d_1,d_2}\right)},\widetilde{\mathcal{L}}\,\underline{\left(Q_{c,i_c,d_1,d_2}\right)}^{*},\widetilde{\mathcal{L}}\,\overline{\left(Q_{c,i_c,d_1,d_2}\right)}\right)$$

$$\widetilde{\mathcal{L}}\left(Q_{c,i_c,d_1,d_2}\right)=\begin{Bmatrix} 0 & x < a \& x > d \\ \widetilde{\mathcal{L}}\,\underline{\left(Q_{c,i_c,d_1,d_2}\right)} & a < x < b \\ \widetilde{\mathcal{L}}\,\underline{\left(Q_{c,i_c,d_1,d_2}\right)}^{*} & b < x < c \\ \widetilde{\mathcal{L}}\,\overline{\left(Q_{c,i_c,d_1,d_2}\right)} & c < x < d \end{Bmatrix} \text{Where}$$

$$\widetilde{\mathcal{L}}\,\underline{\left(Q_{c,i_c,d_1,d_2}\right)} = \alpha\left(\frac{e^{-a\beta} - \left((b-a)\beta + 1\right)e^{-b\beta}}{\beta^2(b-a)}\right)$$

$$\widetilde{\mathcal{L}}\,\underline{\left(Q_{c,i_c,d_1,d_2}\right)}^{*} = \frac{\left((c-b)\alpha\beta + \overline{\alpha}\right)e^{-b\beta} - \left(\left((c-b)\overline{\alpha} + (c-b)\alpha\right)\beta + \overline{\alpha}\right)e^{-c\beta}}{(c-b)\beta^2}$$

$$\widetilde{\mathcal{L}}\,\overline{\left(Q_{c,i_c,d_1,d_2}\right)} = \frac{e^{-d\beta} - \left((d-c)\beta - 1\right)e^{-c\beta}}{\beta^2(d-c)}$$

4. Problem formulation

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Let $\emptyset(t,\alpha)$be the number of *radionuclide in* the sample,μ is the state as decay constant and$d\emptyset(t,\alpha)$*be*number of nuclei that undergo decay in time dt then the nuclear decay equation can be delineatedas[13]

$$\frac{d\emptyset(t,\alpha)}{dt} = -\mu\emptyset(t,\alpha)(1)$$

Where $\emptyset(t,\alpha)$=$\left(\underline{\emptyset(t,\alpha)}, \emptyset^*(t,\alpha), \overline{\emptyset(t,\alpha)}\right)$ is a fuzzy membership grade function of $t \geq 0$ and

$\alpha \in [0\ 1]$ *with tarapozoidal* $(\tau_{b\sim c,d_c,i_c})$ *initial condition*

$$\emptyset(0,\alpha) = \left(\underline{\emptyset(0,\alpha)}, \emptyset^*(0,\alpha), \overline{\emptyset(0,\alpha)}\right) = (\ 3\alpha+1, k, 8-3\alpha)k \in [4\ 5](2)$$

Let $\emptyset(t,\alpha)$ is (ii)-differentiable

Applying the fuzzy Laplace transform both side of equation (1).

$$\widetilde{\mathcal{L}}\left(\frac{\partial\emptyset(t,\alpha)}{\partial t}\right) = \widetilde{\mathcal{L}}\,(-\mu\emptyset(t,\alpha))(3)$$

$$-\emptyset(0,\alpha) \ominus -\beta\widetilde{\mathcal{L}}\,[\emptyset(t,\alpha)] = -\mu\widetilde{\mathcal{L}}\,[\emptyset(t,\alpha)](4)$$

Then the following system is going to be obtained.

$$\beta\widetilde{\mathcal{L}}\left[\underline{\emptyset(t,\alpha)}\right] - \underline{\emptyset(0,\alpha)} = -\mu\widetilde{\mathcal{L}}\,[\underline{\emptyset(t,\alpha)}](5)$$

$$\beta\widetilde{\mathcal{L}}\,[\emptyset^*(t,\alpha)] - \emptyset^*(0,\alpha) = -\mu\widetilde{\mathcal{L}}\,[\emptyset^*(0,\alpha)](6)$$

$$\beta\widetilde{\mathcal{L}}\left[\overline{\emptyset(t,\alpha)}\right] - \emptyset^*\left(\overline{\emptyset(0,\alpha)}\right) = -\mu\widetilde{\mathcal{L}}\,[\overline{\emptyset(0,\alpha)}](7)$$

The initial fuzzy trapezoidal condition changes the equations (5-7) into following system

$$(\beta+\mu)\widetilde{\mathcal{L}}\left[\underline{\emptyset(t,\alpha)}\right] - (3\alpha+1) = \ 0(8)$$

$$(\beta+\mu)\widetilde{\mathcal{L}}\,[\emptyset^*(t,\alpha)] - k = \ 0(9)$$

$$(\beta+\mu)\widetilde{\mathcal{L}}\left[\overline{\emptyset(t,\alpha)}\right] - (8-3\alpha) = \ 0(10)$$

Consider the decay constant $\mu = 1$ and applying the inverse Laplace the above coupled equations (8-9) deduced into

$$\underline{\emptyset(t,\alpha)} = (3\alpha+1)\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(11)$$

$$\emptyset^*(t,\alpha) = k\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(12)$$

$$\overline{\emptyset(t,\alpha)} = (8-3\alpha)\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(13)$$

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Hence

$$\emptyset(t,\alpha)=\begin{Bmatrix}(3\alpha+1)e^{-t} & \alpha\epsilon[0\ 1]\ and\ (3\alpha+1)\in[1\ 3]\\ ke^{-t} & k\in[4\ 5]\\ (8-3\alpha)e^{-t} & \alpha\epsilon[0\ 1]\ and\ (8-3\alpha)\in[5\ 8]\end{Bmatrix}(14)$$

And corresponding fuzzy solution is

$$\emptyset(t,\alpha)=\begin{Bmatrix}\frac{(x-1)}{3}e^{-t} & 1\le x\le 4\\ e^{-t} & 4<x<5\\ \frac{(8-x)}{3}e^{-t} & 5\le x\le 8\end{Bmatrix} \quad (15)$$

Now theequation (1) is considered with the triangular $(\tau_{b,i})$ initial condition and let $\emptyset(t,\alpha)$ is (ii)-differentiable.

Where $\emptyset(t,\alpha)$=$\left(\emptyset^*(t,\alpha),\overline{\emptyset(t,\alpha)}\right)$ is a fuzzy function of $t\geq 0$ and $\alpha\in[0\ 1]$ *with triagular* $(\tau_{b,i})$ *initial condition*

$$\emptyset(0,\alpha)=\left(\emptyset^*(0,\alpha),\overline{\emptyset(0,\alpha)}\right)=(\,k,8-3\alpha)k\in[1\ 5](16)$$

Applying the fuzzy Laplace transform both side of equation (1).

$$\widetilde{\mathcal{L}}\left(\frac{\partial\emptyset(t,\alpha)}{\partial t}\right)=\widetilde{\mathcal{L}}\,(-\mu\emptyset(t,\alpha))(17)$$

$$-\emptyset(0,\alpha)\ominus-\beta\widetilde{\mathcal{L}}\,[\emptyset(t,\alpha)]=-\mu\widetilde{\mathcal{L}}\,[\emptyset(t,\alpha)](18)$$

Then the following system is going to be obtained.

$$\beta\widetilde{\mathcal{L}}\,[\emptyset^*(t,\alpha)]-\emptyset^*(0,\alpha)=-\mu\widetilde{\mathcal{L}}\,[\emptyset^*(0,\alpha)] \quad (19)$$

$$\beta\widetilde{\mathcal{L}}\left[\overline{\emptyset(t,\alpha)}\right]-\emptyset^*\left(\overline{\emptyset(0,\alpha)}\right)=-\mu\widetilde{\mathcal{L}}\,[\overline{\emptyset(0,\alpha)}] \quad (20)$$

The initial fuzzy trapezoidal condition changes the equations (19-20) into following system

$$(\beta+\mu)\widetilde{\mathcal{L}}\,[\emptyset^*(t,\alpha)]-k=\ 0 \quad (21)$$

$$(\beta+\mu)\widetilde{\mathcal{L}}\left[\overline{\emptyset(t,\alpha)}\right]-(8-3\alpha)=\ 0 \quad (22)$$

Consider the decay constant $\mu=1$ and applying the inverse Laplace the above coupled equations (20-22) deduced into

$$\emptyset^*(t,\alpha)=k\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(23)$$

$$\overline{\emptyset(t,\alpha)}=(8-3\alpha)\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(24)$$

Hence

$$\emptyset(t,\alpha) = \begin{cases} ke^{-t} & k \in [1\ 5] \\ (8-3\alpha)e^{-t} & \alpha\epsilon[0\ 1]\ and\ (8-3\alpha) \in [5\ 8] \end{cases} \quad (25)$$

And corresponding fuzzy solution is

$$\emptyset(t,\alpha) = \begin{Bmatrix} 0 & x < 1, x > 8 \\ e^{-t} & 1 < x < 5 \\ \frac{(8-x)}{3}e^{-t} & 5 \le x \le 8 \end{Bmatrix} \quad (26)$$

Similarly the equation (1) is solved with $\alpha - left$ Quadrilateral fuzzy $Q_{c,i_c,d_1,\ d_2}$ initial condition and it was assumed that $\emptyset(t,\alpha)$ is (ii)-differentiable.

Where $\emptyset(t,\alpha)$=$\left(\underline{\emptyset(t,\alpha)}, \underline{\emptyset^*(t,\alpha)}, \overline{\emptyset(t,\alpha)}\right)$ is a fuzzy function of $t \ge 0$ and $\alpha \in [0\ 1] with$ $\alpha^* - left$ Quadrilateral fuzzy number$(Q_{c,i_c,d_1,\ d_2})initial\ condition$

$$\emptyset(0,\alpha) = \left(\underline{\emptyset(0,\alpha \le \alpha^*)}, \underline{\emptyset^*(t,\alpha^* < \alpha \le 1)}, \overline{\emptyset(0,\alpha)}\right) = \left(\frac{30\alpha+8}{8}, \frac{10\alpha}{2}, 8-3\alpha\right) (27)$$

It is clearly observed that the convexity condition of $Q_{c,i_c,d_1,\ d_2}$ satisfy for higher value of α^* if $\alpha^* \ge 0.75$ above number in (27) is convex.

Applying the fuzzy Laplace transform both side of equation (1).

$$\widetilde{\mathcal{L}}\left(\frac{\partial\emptyset(t,\alpha)}{\partial t}\right) = \widetilde{\mathcal{L}}\,(-\mu\emptyset(t,\alpha)) \quad (28)$$

$$-\emptyset(0,\alpha) \ominus -\beta\widetilde{\mathcal{L}}\,[\emptyset(t,\alpha)] = -\mu\widetilde{\mathcal{L}}\,[\emptyset(t,\alpha)] (29)$$

Then the following system is going to be obtained.

$$\beta\widetilde{\mathcal{L}}\left[\underline{\emptyset(t,\alpha)}\right] - \underline{\emptyset(0,\alpha)} = -\mu\widetilde{\mathcal{L}}\,[\underline{\emptyset(t,\alpha)}] \quad (30)$$

$$\beta\widetilde{\mathcal{L}}\left[\underline{\emptyset^*(t,\alpha)}\right] - \underline{\emptyset^*(t,\alpha)} = -\mu\widetilde{\mathcal{L}}\,[\underline{\emptyset^*(t,\alpha)}] (31)$$

$$\beta\widetilde{\mathcal{L}}\left[\overline{\emptyset(t,\alpha)}\right] - \emptyset^*\left(\overline{\emptyset(0,\alpha)}\right) = -\mu\widetilde{\mathcal{L}}\,[\overline{\emptyset(0,\alpha)}] (32)$$

The initial fuzzy $\alpha - left$ quadrilateral fuzzy condition changes the equations (30-32) into following system

$$(\beta+\mu)\widetilde{\mathcal{L}}\left[\underline{\emptyset(t,\alpha)}\right] - \left(\frac{30\alpha+8}{8}\right) = \ 0\ (33)$$

$$(\beta+\mu)\widetilde{\mathcal{L}}\left[\underline{\emptyset^*(t,\alpha)}\right] - \frac{10\alpha}{2} = \ 0 (34)$$

$$(\beta+\mu)\widetilde{\mathcal{L}}\left[\overline{\emptyset(t,\alpha)}\right] - (8-3\alpha) = \ 0 (35)$$

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Consider the decay constant $\mu = 1$ and applying the inverse Laplace the above coupled equations (33-35) deduced into

$$\underline{\emptyset(t,\alpha)} = \left(\frac{30\alpha+8}{8}\right)\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(36)$$

$$\underline{\emptyset^*(t,\alpha)} = \frac{10\alpha}{2}\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(37)$$

$$\overline{\emptyset(t,\alpha)} = (8-3\alpha)\widetilde{\mathcal{L}}^{-1}\left(\frac{1}{1+\beta}\right)(38)$$

Hence

$$\emptyset(t,\alpha) = \begin{Bmatrix} \left(\frac{30\alpha+8}{8}\right)e^{-t} & \alpha\epsilon[0\ 1]\ and\ \left(\frac{30\alpha+8}{8}\right) \in [1\ 4] \\ \frac{10\alpha}{2}e^{-t} & k \in [4\ 5] \\ (8-3\alpha)e^{-t} & \alpha\epsilon[0\ 1]\ and\ (8-3\alpha) \in [5\ 8] \end{Bmatrix}(39)$$

And corresponding fuzzy solution is

$$\emptyset(t,\alpha) = \begin{Bmatrix} \frac{0.8*(x-1)}{3}e^{-t} & 1 \le x \le 4 \\ (0.8+0.2*(x-4))e^{-t} & 4 < x < 5 \\ \frac{(8-x)}{3}e^{-t} & 5 \le x \le 8 \end{Bmatrix} \quad (40)$$

5. Result and analysis

Using the described methodology the solution of nuclear decaying differential equation under all three imposed fuzzy initial condition is obtained. Figure 1 shows the nuclear decay under the trapezoidal and quadrilateral conditions and figure 2 shows the nuclear decay under the triangular condition. In figure 1 three decaying curves represents the lower, central and upper solutions and in figure 2 two decaying curves represents the lower and upper solutions. The figure 3, 4 and 5 shows the respective fuzzy solution under the $(\tau_{b\sim c,d_c,i_c})$, $(\tau_{b,i})$ $and\ \alpha - left\ (Qc,ic,d1,\ \ d2)$ initial condition with varying time and it is observed that radionuclide decreases as time increases and these solutions also can be validated with figure 1 and 2 .It was noticed that the value of $\alpha\ for\ \alpha - left\ (Q_{c,i_c,d_1,\ d_2})$ condition is greater than 0.75 so that $Q_{c,i_c,d_1,\ d_2}$ fulfil the convexity condition of fuzzy number.

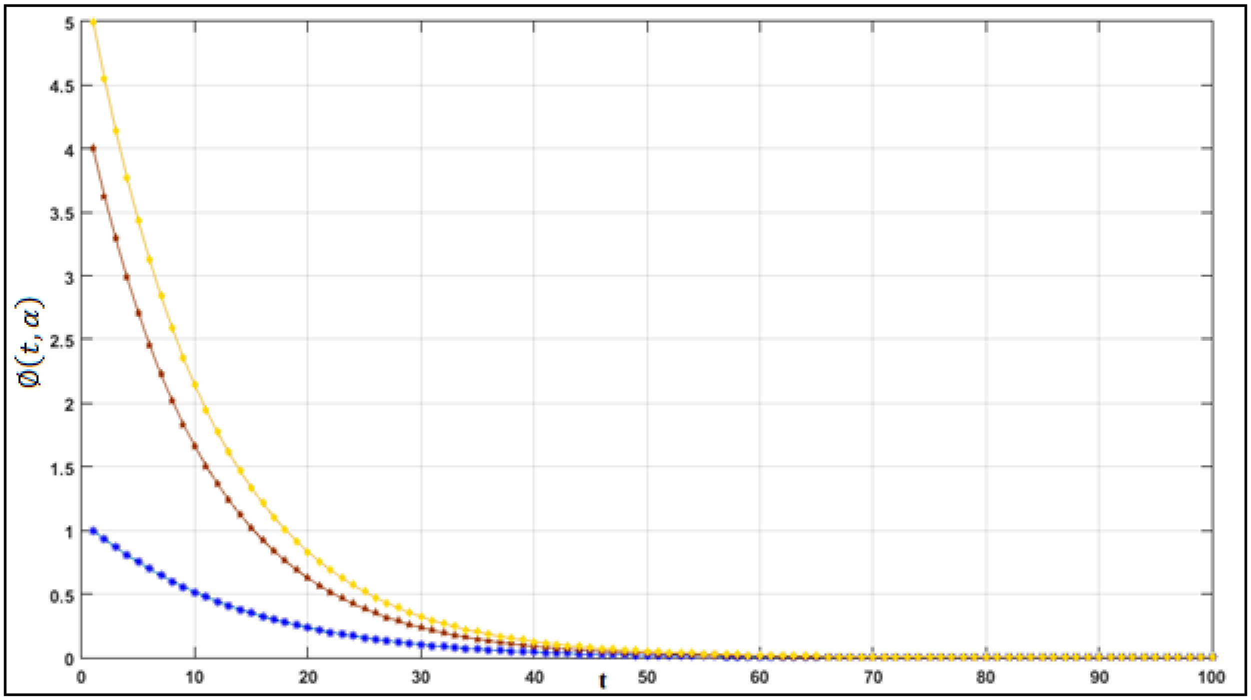

Figure 1: Solution of the nuclear decay equation by using trapezoidal $\left(\tau_{b\sim c,d_c,i_c}\right)$ and quadrilateral fuzzy$(Q_{c,i_c,d_1,\ d_2})$ initial conditions

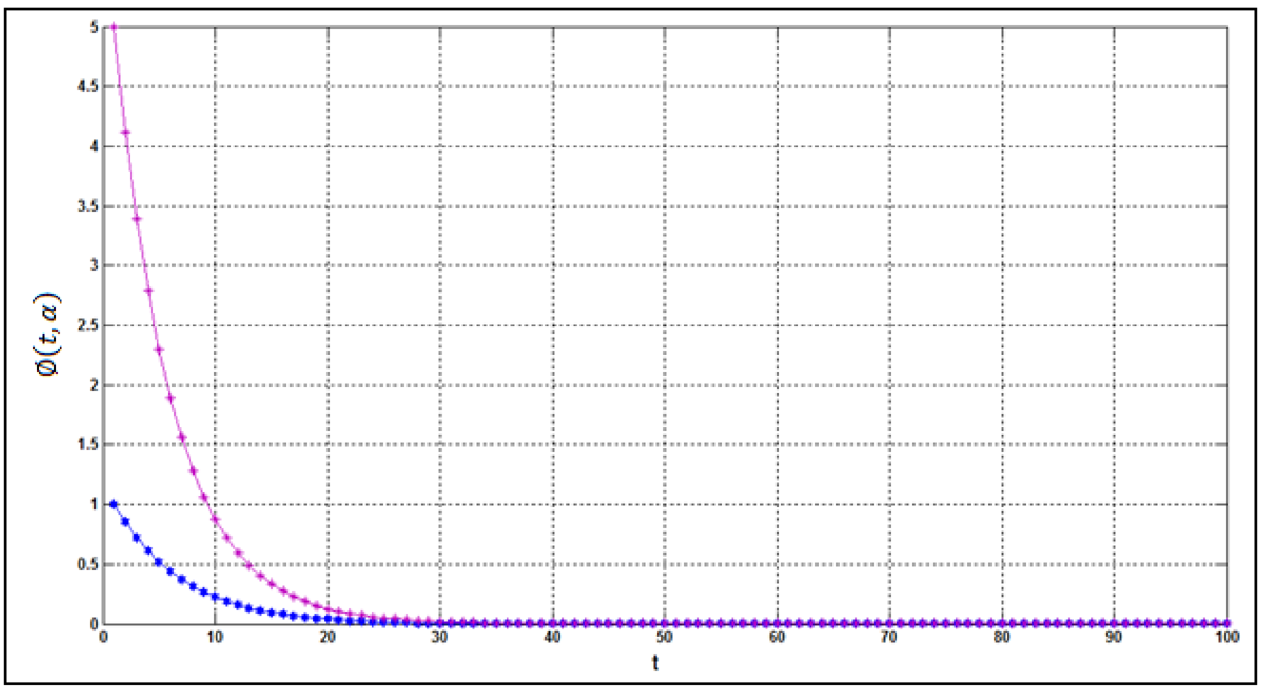

Figure 2:Solution of the nuclear decay equation by using triangular $\left(\tau_{b,i}\right)$ initial condition (16)

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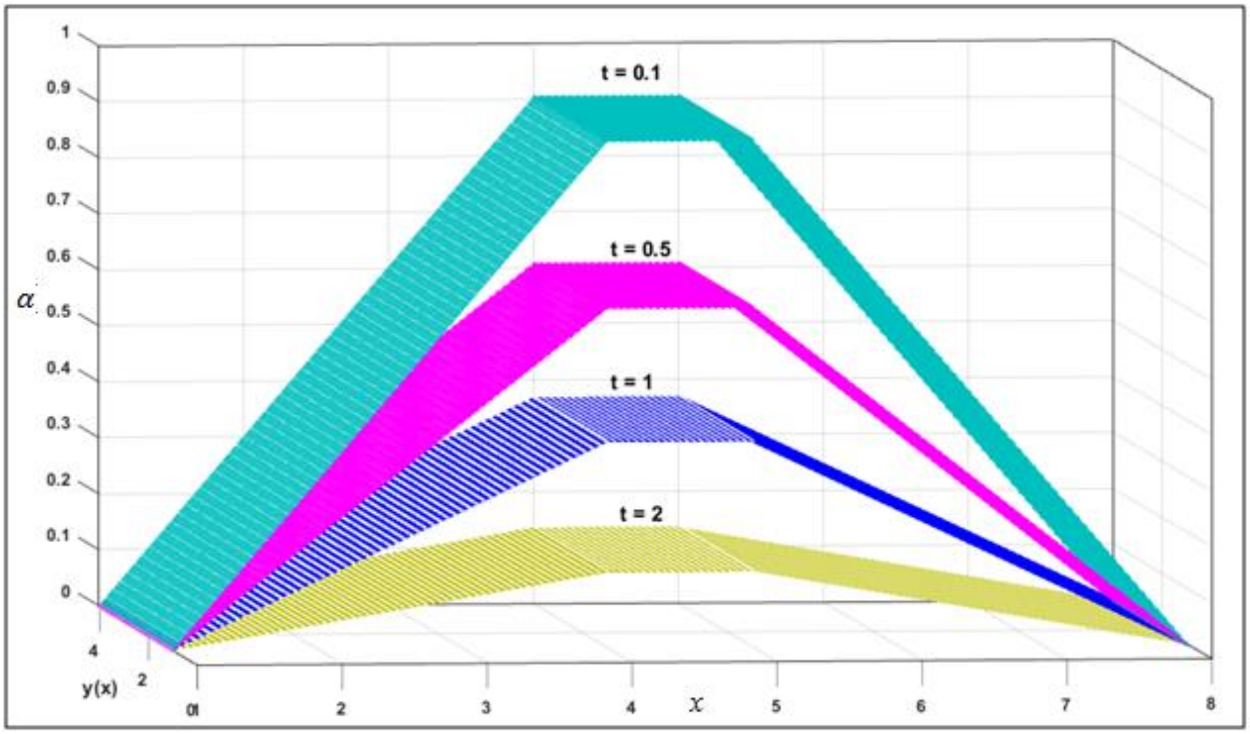

Figure 3:. The fuzzy solution of the nuclear decay equation using trapezoidal $\left(\tau_{b\sim c,d_c,i_c}\right)$ initial condition (2)

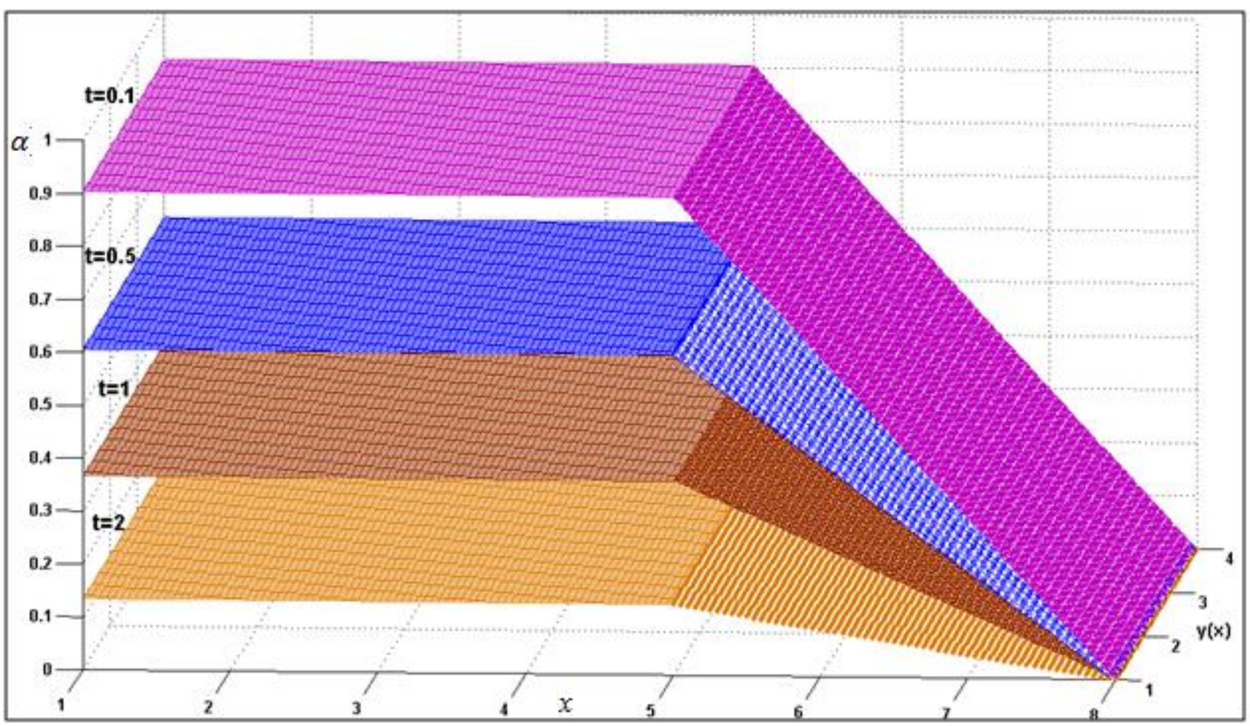

Figure 4:. The fuzzy solution of the nuclear decay equation using triangular $\left(\tau_{b,i}\right)$ initial

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Condition (16)

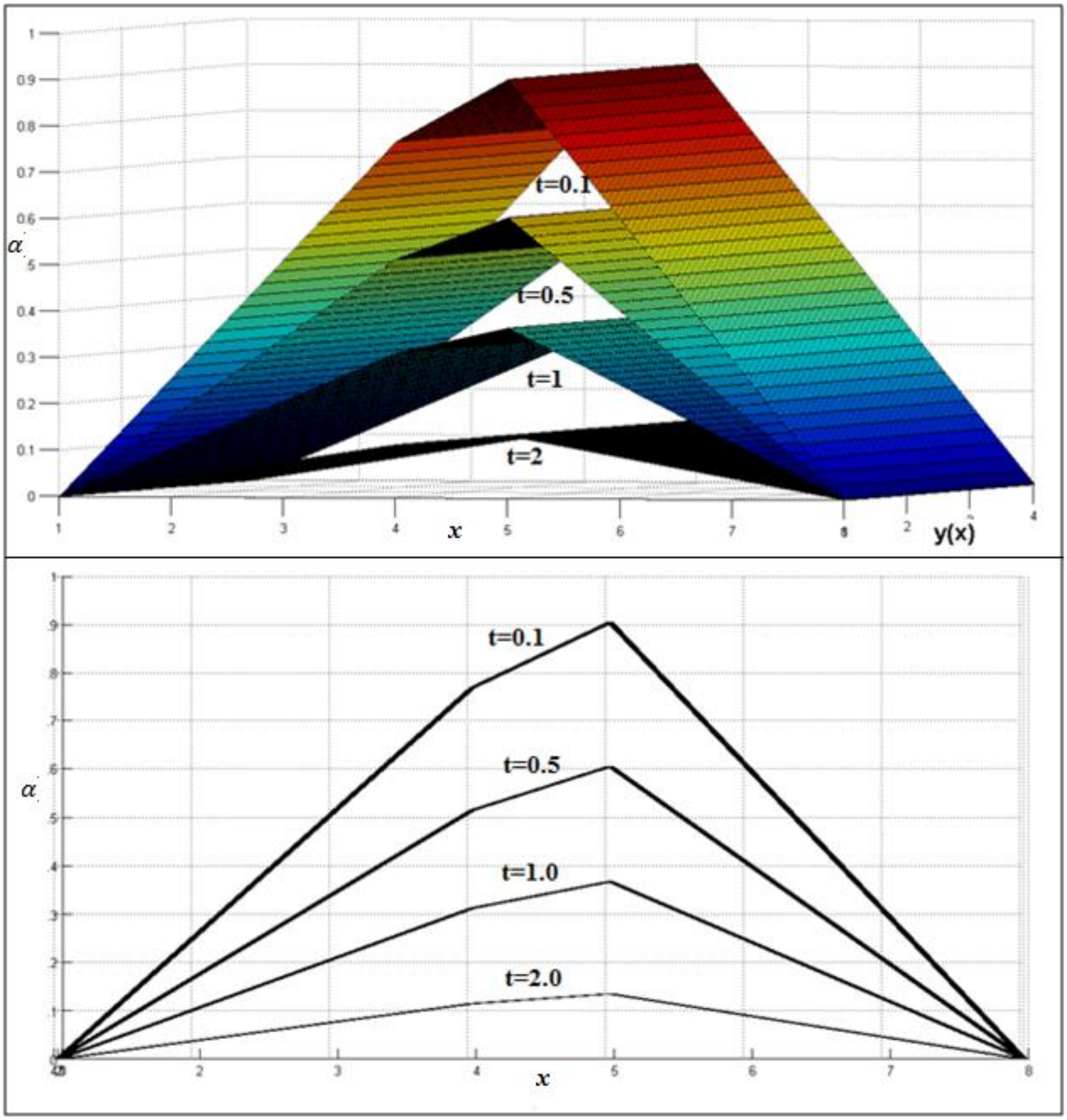

Figure 5: The fuzzy solution of the nuclear decay equation using $\alpha - left$ quadrilateral fuzzy$(Q_{c,i_c,d_1,\ d_2})$ initial condition (16).

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Conclusion

In this paper, the radioactive decaying differential equation under the*trapezoidal*$\left(\tau_{b\sim c,d_c,i_c}\right)$ *,right angled triangular*$\left(\tau_{b,i}\right)$ $and\ \ \alpha - left\, quadrilateral(Q_{c,i_c,d_1,\ d_2})$ initial conditions has formulated and solve by fuzzy Laplace transform. A new $\alpha - left\, quadrilateral(Q_{c,i_c,d_1,\ d_2})$fuzzy number as initial condition is introduced first time to explore the FDEs. The lower central and upper solutions according the applied fuzzy initial conditions are presented. The respective fuzzy solutions under all three imposed conditions have presented. The following remarks can be concluded

- The number of radionuclide decreases as time increases and it vanishes with higher time.
- The greater value of $\alpha say \geq 0.75\ for\ \alpha - left\, quadrilateral(Q_{c,i_c,d_1,\ d_2})$ provides the better convexity.
- The differential equation can be explored with some other $\alpha - left$ and $right$ poly fuzzy number.

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