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A Note on Generalised Pre Regular Weakly (Gprw)-Connected and Compact Sets In Topological Spaces

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Abstract

Connected and compact sets play an important role in the topology and there are many properties of it which cannot be studied without these two concepts. For that reason in this paper a generalised form of connected and compact sets are inaugurated, titled as $gprw$-connected and $gprw$-compact sets. Such contemporary type of sets encircled by rw-connected and gpr-compact sets. So in this present research some significant results which are in the form of theorems, lemmas and counter examples are considered under study.

Introduction

Kutatowiski closure axioms play an important role to define the topological spaces. So one can envision that, closed set is a significant idea in the topology. Initially Levine [1] investigated the generalised closed set in 1970. This idea has been contemplated widely in the ongoing years by numerous topologists in light of the fact that generalised closed set are not just regular speculation of closed sets. Additionally, they recommend a few modernprinciples of topology theory. A large portion of these modernprinciples areweaker than T_1 separation axioms, few of these are found very helpful in software engineering and computerized topology.Other new properties are characterized by a variety of the property of submaximality. Besides, the investigation of thisnew sets additionally gives a new portrayal few familiar classes of spaces by Caw et al. [2].Gpr-closed set is proposed by Gnanambal [3] in 1997 and further the concept of preregular$T_{\frac{1}{2}}$space and gpr-continuity was presented. Benchalli and Wali [4] proposed regular weakly closed set which is encircled by w-closed [5] set and regular generalised closed sets [6]. Also they discussed some important properties of it. In the present paper a generalised form of connected and compact sets are inaugurated, titled as $gprw$-connected and $gprw$-compact sets. Such contemporary type of sets encircled by rw-connected and gpr-compact sets. So in this present research some significant results which are in the form of theorems, lemmas and counter examples are considered under study

Preliminary Notes

In this paper the following definitions are used

1. $R \subseteq U$is generalized closed (g-closed) [1] set if and only if $cl(R)$ is sub set of O at any time$R$ is subset of O and O is open.
2. $R \subseteq U$is regular open (r-open) [7] set if $R = int(cl(R))$ and regular closed (r-closed) [7] set if $R = cl(int(R))$.
3. $R \subseteq U$ispre-open set [8] if R subset of $int(cl(R))$ and pre-closed [8] set if $cl(int(R))$ subset of R.
4. $R \subseteq U$isregular semiopen [9] for regular open set $O \subset R$ and R is subset of $cl(O)$.
5. $R \subseteq U$isgprw-closed [10] if $pcl(R)$ subset of O, at any time$R$is subset of O and O is regular semi open.
6. Any set of (U, θ) is called
    a. GPR-closed [3] if $pcl(R)$ subset of O whenever R subset of O and O is regular open in U.
    b. $\pi - g -$closed [11]if $cl(R)$ subset of O at any time$R$ subset of O and $O\ is\ \pi - open\ in\ U$.
    c. w-closed [12] if $cl(R)$ subset of O at any time$R$ subset of O and O is semiopen in U.
    d. RWG-closed [13] if $cl(int(R))$ subset of O at any time$R$ subset of O and O is regular open in U.
    e. RW-closed) [4] if $cl(R)$ subset of O at any time$R$ subset of O and O is regular semiopen in U.

Theorem 2.1[14]:$\{\forall\ R \in U{:}\ R\ is\ regular\ semi\ open\}$*is semiopen but not conversely.*

Theorem2.2[14]: *InU, the clopen, $r -$ closed, and$r -$ open sets are regular semiopen.*

3. GPRW-Connected Space in Topological Space:

In this section $gprw$-connectedness is defined and it is shown that a proper subset of a $gprw$-connected space both $gprw$-open and $gprw$-closed. We have also proved that under surjective$gprw$-continuous map the image of $gprw$-connected set is connected.

Defination 3.1: *Topological space(U,θ) called$gprw$-connected if U cannot be described as the disjoint union of two non-empty $gprw$-open sets. And $R \subset U$ is $gprw\text{-}connected\ if\ it\ is$ $gprw$-connected as a subspace.*

Theroem 3.2:*In (U,θ) following statements are equivalent*

1. *$U$is a $gprw$-connected space.*

2. Any subset of U which isgprw-closed and gprw-open are eitherϕor$U$.

3. Every$gprw$-continuous mapping of U to$V$ with two or more points is constant mapping.

Proof: $(1 \to 2)$ Consider$D$which is gprw-closed and opensubset of Uthen $U \setminus D$ is also the same and $U = D \cup (U \setminus D)$ that means we can write U as union of two non-empty sets which is both $gprw$closed and open, but it is the contradiction to the statement that U is $gprw$-connected. Hence either $D = U$ or $D = \phi$.

$(2 \to 1)$ Now suppose U is union of $S$and R, where $S \cap R$ is empty, $S \neq \phi$, $R \neq \phi$ and S, R are $gprw$-open. Since $R = U \setminus S$ is gprw-closed, but by assumption either $R = \phi$or $R = U$, which is the contradiction. Hence (1) is true.

$(2 \to 3)$ Let $t: U \to V$ is a gprw-continuous mapping. Where V is discrete set containing two or more points then $t^{-1}(V)$ is $gprw$-closed and $gprw$-open for each v belongs to V.$U = \bigcap \{t^{-1}(V): v \in V\}$.Now consider$t^{-1}(V) = \phi$or U. If $t^{-1}(V) = \phi$ for all $v \in V$, then t will not be a function. Also there cannot exist more than one $v \in V$ such that $t^{-1}(v) = U$ and$t^{-1}(v') = \phi$where $v \neq v' \in V$. Hence t is constant mapping.

$(3 \to 2)$ Let D be a gprw-open and $gprw$-closed in Usuppose $D \neq \phi$. Let $t: U \to V$ be a gprw-continuous function defined by $t(U \setminus D) = y$ and $t(D) = x$ where x and y are different and x, y belongs to V. By assumption, t is constant, therefore $D = U$.

Theorem 3.3:*Every $gprw$-connected topological space is connected but not conversely.*

Proof: Assume U be a $gprw$-connected space and want to show that U is connected space, we use contradiction proofing to prove this, for that assume$U$ is disconnected that means there exist non-void set R (say) in$U$, which is closed and open both such that $U = R \cup R'$, as R is given to be closed and by theorem 2.2, R is $gprw$-closed then its complement is $gprw$-open also $U = R \cup R'$ that means we are in the position to find out a non-void subset of U which is both $gprw$-closed and gprw-open. Which shows that U is not a $gprw$-connected and it is a contradicting to our consideration. Hence U is connected.

Converse of the above theorem is not true that is every connected space is not $gprw$-connected that can be seen from the following example.

Example 3.4: Let a topological space $U = \{i, j, k\}$ with topology $\theta = \{\phi, U, \{i\}, \{i, j\}\}$ then we have the collection of all $gprw$-closed sets in $X$is $\{\phi, U, \{i, k\}, \{j, k\}, \{i, j\}, \{k\}, \{j\}, \{i\}\}$. So clearly given U is not gprw-connected as we have a non-voidset $\{i, j\}$ of U which is$gprw$-closed and $gprw$-open both. But the given topological space U is connected as we have the both closed and open subset of U are ϕ and U itself.

Theorem 3.5: *All$gprw$-connected is w-connected topological space (where w-connected space is if it cannot be written as union of two non empty proper weakly open sets), but not conversely.*

Proof: Assume$U$a $gprw$-connected space and want to show that U is w-connected space, we use contradiction proofing to prove this, for that assume$U$ is w-disconnected that means there exist some non-void set$R$ (say) of U, which is w-closed and open such that $U = R \cup R'$, as R is given to be w-closed.$R$is$gprw$-closed then its complement is $gprw$-open also $U = R \cup R'$ that means we are in the position to find out a non-void set of U which is$gprw$-closedand$gprw$-open. Which shows that U is not a $gprw$-connected and it is a contradicting to our consideration. Hence U is w-connected.

Theorem 3.6: *All$gprw$-connected is rw-connected topological space (where rw-connected space is if it cannot be written as union of two non empty $proper$ rw-open sets).*

Proof: Let U be a $gprw$-connected space and we want to prove that U is rw-connected space, we use contradiction proofing to prove this, So suppose U is rw-disconnected that means there exist some non-void set R (say) in U, which is rw-closed and rw-open such that $U = R \cup R'$, as R is given to be rw-closed and by theorem 2.1.R is $gprw$-closed then its complement is $gprw$-open also $U = R \cup R'$ that means we are in the position to find out a non-void set in$U$ which is $gprw$-closed and $gprw$-open. Which shows that $U$is $gprw$-disconnected and it is a contradicting to our considerationthat U is $gprw$ $-$connected. Hence U is rw-connected.

Theorem 3.7: *Let U and V be to topological space, then*

- a. *If $p: U \to V$ is a gprw-continuous and onto where U is gprw-connected then V is connected.*
- b. *If $p: U \to V$ is a gprw-irresolute and onto where U is gprw-connected then V is gprw-connecte*

Proof:

- a. We prove this result by contradiction for that Suppose V is not connected. Then V can be written as union of S and R, where $S \cap R$ is empty, $S \neq \phi$, $R \neq \phi$ and S and $R$are open in V. Since p is gprw-continuous and onto therefore $U = p^{-1}(S) \cup p^{-1}(R)$ where $p^{-1}(S)$ and $p^{-1}(R)$ are disjoint non-voidgprw-open subset of U it means,$U$is $gprw$-disconnected which is contradiction. Hence V is connected.
- b. Assume the contrary, suppose V is not$gprw$-connected.Then V can be written as union of S and R, where $S \cap R$ is empty, $S \neq \phi$, $R \neq \phi$and R and $S$$gprw$-open in V. Since p is gprw-irresolute and onto therefore $U = p^{-1}(S) \cup p^{-1}(R)$ where $p^{-1}(S)$ and $p^{-1}(R)$ are disjoint non-void$gprw$-open subset of U it means,$U$is $gprw$-disconnected which is a contradiction. Hence V is $gprw$-connected.

4. $GPRW$-Compact Space in Topological Space

In this section we shall propose the definition of $gprw$-compact space, gprw-open cover and also some results based on them.

Definition 4.1:*In (U, θ)topology, A collection $\{R_j : j \in \Delta\}$ of $gprw$-open sets is called a $gprw$-open cover of D if $D \subset \bigcup \{R_j : j \in \Delta\}$ holds.*

Definition 4.2:*(U, θ) which is a topological space, iscalled $gprw$-compact if each$gprw$-open cover of U has a sub cover which is also finite.*

Definition 4.3:*In (U,θ)topology, a set $D\subset U$is called $gprw$-compact relative to U, if for each collection $\{R_j : j\in\Delta\}$ of $gprw$-open subsets of U such that $D\subset\bigcup\{R_j : j\in\Delta\}$ then $\exists$ a finite subset Δ_0 of Δ such that $D\subset\bigcup\{R_j : j\in\Delta_0\}$.*

Definition 4.4:*In (U,θ)topology, a set $D\subset U$is called $gprw$-compact if D is$gprw$-compact as a subspace of U.*

Theorem 4.5: *A $gprw$-closed subset of a $gprw$-compact topological space $U$is $gprw-compact\ relative\ to\ U$.*

Proof: Assume$R$ be a $gprw$-closed in U which is $gprw$-compact space. Then $U\setminus R$ is $gprw$-open. Let Ω be a $gprw$-open cover for R thengprw-open cover of Uwhich is $\{\Omega, U\setminus R\}$. AsU is gprw-compact therefore it contain a finite sub cover,$\{P_1,P_2,\dots,P_n\}=\Omega_1(say)$. If $U\setminus R$ does not belong to Ω then Ω_1 is a finite sub cover of R. If $U\setminus R\in\Omega_1$ then $\Omega_1\setminus(U\setminus R)$ is a sub cover of R. Hence R is $gprw$-compact relative to U.

Theorem 4.6: *Let $p: U\to V$ be a surjective and gprw-continuous function where U is gprw-compact then$V$ is compact.*

Proof: Assume$\{R_j : j\in\Delta\}$ open cover of V. Now p is given to be gprw-continuous therefore $\{p^{-1}(R_j): j\in\Delta\}$ is a $gprw$-open cover of U. Since U is $gprw$-compact, so we must have finite sub cover say $\{p(R_1), p^{-1}(R_2),\dots,p^{-1}(R_n)\}$. And p is surjective shows that V has open cover as$\{R_1,R_2,\dots,R_n\}$ so hence V is compact.

Theorem 4.7: *If $t: U\to V$ is a gprw-irresolute mapping and $D\subset U$ is gprw-compact relative to U then the image $t(D)$ is gprw-compact relative to V.*

Proof: Assume$\{R_j : j\in\Delta\}$ be a collection of $gprw$-open sets in V such that $t(D)\subset\{R_j : j\in\Delta\}$. Then $D\subset\backslash\{t^{-1}(R_j) : j\in\Delta\}$ where $t^{-1}(R_j)$ is $gprw$-open in U for each j, since D is $gprw$-compact relative to U there exist a finite sub collection $\{R_1,R_2,\dots,R_n\}$ such that $D\subset\{t^{-1}(R_j): j=1,2,\dots,n\}$ that is $t(D)\subset\{R_j : i=1,2,\dots,n\}$. Hence $t(D)$ is gprw-compact relative to V.

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