

Impact of Viscous Liquid Loadings on Shear Horizontal Waves In An Isotropic Layer Lying Over Microstructural Substrate

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Abstract: The paper aims to study about the propagation behavior of shear horizontal type waves in a homogeneous isotropic layer sandwiched between viscous liquid layer and a microstructural substrate which is modeled using consistent couple stress theory. Consistent couple stress theory involves an additional material parameter called characteristic length of material to capture the role of internal microstructures of material. Dispersion and damping equations are calculated for the propagation of shear horizontal waves through the considered geometry using interfacial boundary conditions. The impact of various parameters such as width ratio of the layers, viscosity and density of viscous liquid layer and characteristic length parameter of consistent couple stress substrate are studied on the phase and damping velocities of shear horizontal type waves.

Key Words: Shear horizontal waves, Consistent couple stress theory, Viscous liquid loadings, Characteristic length, Isotropic material

1. Introduction

Shear horizontal waves are surface waves which are polarized in a plane determined by normal to the direction of propagation of wave. Study of these waves is crucial as it finds numerous applications in various fields such as seismology, geomechanics, earthquake engineering, non-destructive testing techniques. Researchers have studied about these waves considering various geometries such as Kakar [8], Sharma and Kumar [12], Singh *et al.* [1]. These type of waves are also utilized in surface waves acoustic devices (SAW) to find material flaws (Piliposian *et al.* [4], Ma *et al.* [7]) and are also used in biological biosensors to find human diseases such as Ten *et al.* [9].

Consistent couple stress theory (Hadjefandiari, Dargush [2]) is a generalized microcontinuum theory involving three material parameters (μ, λ, η) , out of which two (μ, λ) are Lamé's parameters and third (η) is called as couple stress coefficient. Couple stress coefficient depends upon a characteristic length parameter (l) through a mathematical relation $(\eta = \mu l^2)$. Characteristic length is of the order of average material cell size and it is responsible for bringing the role of internal microstructures of material into considerations. Researchers have utilized consistent couple stress theory to study various problems such as [10-11].

Here in this paper, we have studied the propagation of shear horizontal type waves in a homogeneous isotropic layer lying over a microstructural substrate, which is modeled using

consistent couple stress theory and isotropic layer is loaded with a finite thicker viscous liquid layer. As shear horizontal waves or Love type waves are used in SAW devices which are utilized in such situations where these are immersed in viscous liquid such as Cui *et al.* [6], Nie *et al.* [3], Baroi *et al.* [5]. The impact of various parameters involved in the problem such as ratio of width of two layers, viscosity parameter, density of liquid layer and characteristic length of substrate are studied on the behavior of shear horizontal waves.

2. Formulation and solution of the problem

Consider an isotropic homogeneous layer of finite thickness which is sandwiched between a finite thicker viscous liquid layer and a microstructural half space governed by consistent couple stress theory. The origin of the cartesian coordinate system $O(x, y, z)$ lies at the interfacial surface joining isotropic layer and couple stress substrate. The wave is considered to propagate in the direction of x -axis and y -axis is positive going downwards into the substrate. The free surface of viscous liquid layer is at $y = -(H_1 + H_2)$. The interface surface between viscous liquid layer and isotropic elastic layer is given by $y = -H_2$ and the interfacial surface between isotropic elastic layer and couple stress substrate is given at $y = 0$. For the propagation of Love type waves in the considered geometry, we assumed that displacement components are independent of z coordinate, that is $\frac{\partial}{\partial z} \equiv 0$, so if (u, v, w) are the displacement components of a point in any medium, then $u = v = 0$ and w is function of parameters x, y and t .

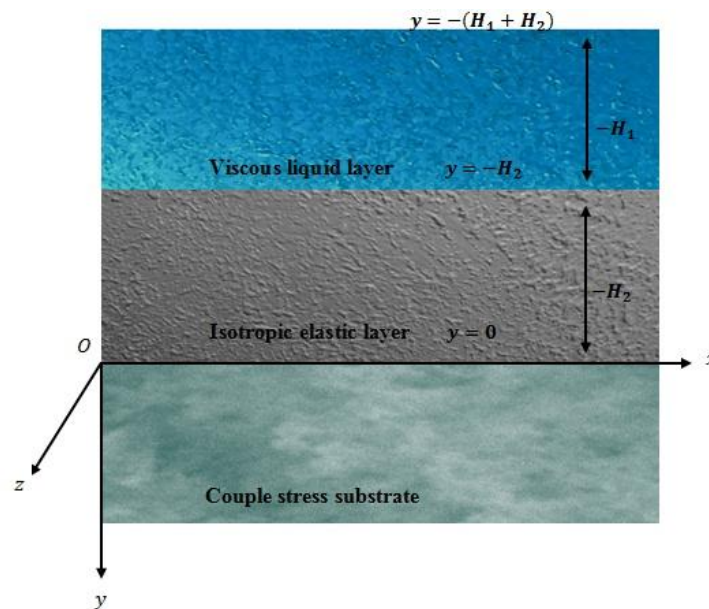


Figure 1: Geometry of the problem

2.1 Solution of viscous liquid layer

The viscous liquid layer is assumed to be placed over an isotropic elastic layer which is resting over a couple stress half space. The equation of motion for the viscous liquid layer is given as [5]

$$\mu_1 \left(\frac{\partial^2 w_l}{\partial x^2} + \frac{\partial^2 w_l}{\partial y^2} \right) = \rho_1 \frac{\partial w_l}{\partial t} \tag{1}$$

where the symbols ρ_1 , μ_1 and w_l represent the liquid mass density, viscous coefficient of liquid and velocity of liquid particle along z-direction, respectively. Let us assume solution for the above differential equation as

$$w_l = p(y)e^{ik(x-ct)} \tag{2}$$

where k is wave number, c is the phase velocity. Substituting above solution into equation (2), we get

$$\frac{d^2 p(y)}{dy^2} - \epsilon^2 p(y) = 0 \tag{3}$$

where $k^2 - \frac{ikc\rho_1}{\mu_1} = k^2 - \frac{ikc}{c_1^2} = \epsilon^2$ and $\frac{\rho_1}{\mu_1} = \frac{1}{c_1^2}$

The solution of equation (3) is given as

$$p(y) = A_1 e^{\epsilon y} + B_1 e^{-\epsilon y}, \text{ where } A_1 \text{ and } B_1 \text{ are arbitrary constants} \tag{4}$$

$$\text{Hence } w_l = (A_1 e^{\epsilon y} + B_1 e^{-\epsilon y}) e^{ik(x-ct)} \tag{5}$$

The only non-vanishing shear stress is given as

$$(\tau_{yz})_{vl} = \mu_1 \frac{\partial w_l}{\partial y} = \mu_1 (A_1 \epsilon e^{\epsilon y} - B_1 \epsilon e^{-\epsilon y}) e^{ik(x-ct)} \tag{6}$$

2.2 Solution of intermediate isotropic layer

The equation of motion for an isotropic homogeneous elastic layer under the conditions needed for the propagation of Love waves $\vec{u} = (0, 0, w_2)$ and $\frac{\partial}{\partial z} \equiv 0$ can be written as

$$\mu_2 \nabla^2 w_2 = \rho_2 \frac{\partial^2 w_2}{\partial t^2} \tag{7}$$

where the symbols ρ_2 , μ_2 and w_2 represent density, Lamé’s constant and displacement of the elastic material particle along z-direction, respectively.

Let the solution for the equation (7) be $w_2 = g(y)e^{ik(x-ct)}$, so from equation (7), we get

$$\frac{d^2 g(y)}{dy^2} + k^2 \left(\frac{c^2}{c_2^2} - 1 \right) g(y) = 0, \text{ where } \frac{\rho_2}{\mu_2} = \frac{1}{c_2^2} \tag{8}$$

The solution of equation (8) is given as

$$g(y) = A_2 \cos(\sigma y) + B_2 \sin(\sigma y) \tag{9}$$

where $k^2 \left(\frac{c^2}{c_2^2} - 1 \right) = \sigma^2$, A_2 and B_2 are arbitrary constants

$$\text{Hence, } w_2 = ((A_2 \cos(\sigma y) + B_2 \sin(\sigma y)) e^{ik(x-ct)}) \tag{10}$$

Constitutive relation for homogeneous isotropic layer is given as

$$S_{ij} = \lambda_2 u_{k,k} \delta_{ij} + 2\mu_2 e_{ij}, \text{ where } e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

The only non-vanishing stress component is given as

$$S_{yz} = \mu_2 \frac{\partial w_2}{\partial y} = \mu_2 (-\sigma A_2 \sin(\sigma y) + \sigma B_2 \cos(\sigma y)) e^{ik(x-ct)} \tag{11}$$

2.3 Solution of consistent couple stress half space

The equation of motion for consistent couple stress theory for isotropic material is given as (Hadjefandiari and Dargush [2])

$$(\lambda_3 + \mu_3 + \eta \nabla^2) \nabla(\nabla \cdot \vec{u}) + (\mu_3 - \eta \nabla^2) \nabla^2 \vec{u} = \rho_3 \frac{\partial^2 \vec{u}}{\partial t^2} \tag{12}$$

where λ_3 and μ_3 are Lamé's constants, ρ_3 is the density of material of the couple stress substrate, $\eta = \mu_3 l^2$ is couple stress coefficient, l is characteristic length, and $\vec{u} = (u_3, v_3, w_3)$ is the displacement vector. Now, using the conditions $u_3 = v_3 = 0$ and $\frac{\partial}{\partial z} \equiv 0$. Equation (12) reduces to

$$\begin{aligned} (\mu_3 - \eta \nabla^2) \nabla^2 w_3 &= \rho_3 \frac{\partial^2 w_3}{\partial t^2} \\ \left(\frac{\partial^2 w_3}{\partial x^2} + \frac{\partial^2 w_3}{\partial y^2} \right) - l^2 \left(\frac{\partial^4 w_3}{\partial x^4} + 2 \frac{\partial^4 w_3}{\partial y^2 \partial x^2} + \frac{\partial^4 w_3}{\partial y^4} \right) &= \frac{1}{c_3^2} \frac{\partial^2 w_3}{\partial t^2} \end{aligned} \tag{13}$$

where $\eta = \mu_3 l^2$ and $\frac{\rho_3}{\mu_3} = \frac{1}{c_3^2}$

Let the solution for the equation (13) be $w_3 = f(y) e^{ik(x-ct)}$, where k is wave number and c is phase velocity, so from equation (13), we get

$$\begin{aligned} \frac{d^4 f(y)}{dy^4} - P \frac{d^2 f(y)}{dy^2} + Q f(y) &= 0 \\ \text{where } \left(2k^2 + \frac{1}{l^2} \right) &= P \text{ and } \left(\frac{k^2}{l^2} + k^4 - \frac{k^2 c^2}{l^2 c_3^2} \right) = Q \end{aligned} \tag{14}$$

Since in the couple stress substrate amplitude of wave must decrease with increase in depth, so the solution of the equation (14) is given as

$$f(y) = A_3 e^{-a_2 y} + B_3 e^{-b_2 y} \text{ where } A_3 \text{ and } B_3 \text{ are arbitrary constants}$$

$$\text{where } a_2 = \sqrt{\frac{P + \sqrt{P^2 - 4Q}}{2}} \text{ and } b_2 = \sqrt{\frac{P - \sqrt{P^2 - 4Q}}{2}}$$

$$\text{Hence, } w_3 = (A_3 e^{-a_2 y} + B_3 e^{-b_2 y}) e^{ik(x-ct)} \tag{15}$$

The consecutive relations for consistent couple stress theory are given as

$$\sigma_{ji} = \lambda_3 u_{k,k} \delta_{ij} + \mu_3 (u_{i,j} + u_{j,i}) - \eta \nabla^2 (u_{i,j} + u_{j,i}) \tag{16}$$

$$\mu_{ji} = 4\eta (\omega_{i,j} - \omega_{j,i}) \text{ where } \omega_i = \frac{1}{2} \epsilon_{ijk} u_{k,j} \tag{17}$$

Here u_i are the displacement components, σ_{ji} is non-symmetric force-stress tensor, μ_{ji} is skew-symmetric couple-stress tensor, δ_{ij} is Kronecker's delta and ϵ_{ijk} is permutation tensor and $i, j, k = 1, 2, 3$.

$$\sigma_{yz} = \mu_3 \frac{\partial w_3}{\partial y} + \mu_3 l^2 \left(\frac{\partial^3 w_3}{\partial x^2 \partial y} + \frac{\partial^3 y}{\partial y^3} \right)$$

$$\sigma_{yz} = \left[\mu_3 (-a_2 A_3 e^{-a_2 y} - b_2 B_3 e^{-b_2 y}) + \mu_3 l^2 (-a_2 A_3 k^2 e^{-a_2 y} - b_2 B_3 k^2 e^{-b_2 y} + a_2^3 A_3 e^{-a_2 y} + b_2^3 B_3 e^{-b_2 y}) \right] e^{ik(x-ct)} \tag{18}$$

$$\mu_{xy} = -2\eta \left(\frac{\partial^2 w_3}{\partial x^2} + \frac{\partial^2 w_3}{\partial y^2} \right)$$

$$\mu_{xy} = 2\mu_3 l^2 (k^2 A_3 e^{-a_2 y} + k^2 B_3 e^{-b_2 y} - a_2^2 A_3 e^{-a_2 y} - b_2^2 B_3 e^{-b_2 y}) e^{ik(x-ct)} \tag{19}$$

3. Boundary conditions

Boundary conditions-I

(i) Top surface of viscous liquid layer should be stress free, that is

$$(\tau_{yz})_{vl} = 0 \text{ at } y = -(H_1 + H_2)$$

Boundary conditions-II

(ii) Continuity of velocity components at the interface $y = -H_2$ that is $w_l = \frac{\partial w_2}{\partial t}$, at $y = -H_2$

(iii) Continuity of the stress components at $y = -H_2$ that is $(\tau_{yz})_{vl} = S_{yz}$, at $y = -H_2$

Boundary conditions-III

(iv) Displacement components should be continuous at the interface $y = 0$ that is $w_2 = w_3$, at $y = 0$

(v) Continuity of the stress components at $y = 0$ that is $S_{yz} = \sigma_{yz}$ at $y = 0$

(vi) The couple stress tensor must vanish at $y = 0$ that is $\mu_{xy} = 0$ at $y = 0$

Using above stated boundary conditions, we get following system of six homogeneous linear equations

$$A_1 e^{-\epsilon(H_1+H_2)} - B_1 e^{\epsilon(H_1+H_2)} = 0 \tag{20}$$

$$A_1 e^{-\epsilon H_2} + B_1 e^{\epsilon H_2} - ikccos(\sigma H_2)A_2 + ikcsin(\sigma H_2)B_2 = 0 \tag{21}$$

$$\mu_1 \epsilon e^{-\epsilon H_2} A_1 - \mu_1 \epsilon e^{\epsilon H_2} B_1 - \mu_2 \sigma sin(\sigma H_2)A_2 - \mu_2 \sigma cos(\sigma H_2)B_2 = 0 \tag{22}$$

$$A_2 - A_3 - B_3 = 0 \tag{23}$$

$$\mu_2 \sigma B_2 + (\mu_3 a_2 + \eta k^2 a_2 - \eta a_2^3)A_3 + (\mu_3 b_2 + \eta k^2 b_2 - \eta b_2^3)B_3 = 0 \tag{24}$$

$$(k^2 - a_2^2)A_3 + (k^2 - b_2^2)B_3 = 0 \tag{25}$$

For non-trivial solution of above mentioned system of homogeneous linear equations, determinant of coefficient matrix for unknowns A_1, B_1, A_2, B_2, A_3 and B_3 must vanish, so by solving the determinant, we obtain following equation

$$R_2 T_{10} - i I_2 T_{10} + \mu_1 (i R_1 R_3 + R_1 I_3 + I_1 R_3 - i I_1 I_3) T_{11} = 0 \tag{26}$$

From real part of equation (26), we get dispersion equation for Love type waves as

$$R_2 T_{10} + \mu_1 (R_1 I_3 + I_1 R_3) T_{11} = 0 \tag{27}$$

and from imaginary part of equation (26), we get damping equation as

$$I_2 T_{10} + \mu_1 (I_1 I_3 - R_1 R_3) T_{11} = 0 \tag{28}$$

where $\epsilon = \sqrt{k^2 - \frac{ikc\rho_1}{\mu_1}} = R_1 - iI_1$, $R_1 = \sqrt{r} \cos\left(\frac{\theta}{2}\right)$, $I_1 = \sqrt{r} \sin\left(\frac{\theta}{2}\right)$, $r = \sqrt{k^4 + \frac{k^2 c^2}{c_1^4}}$,

$$\theta = \tan^{-1}(c/kc_1^2), R_2 = 2 \cosh(R_1 H_1) \cos(I_1 H_1), I_2 = 2 \sinh(R_1 H_1) \sin(I_1 H_1),$$

$$R_3 = 2 \sinh(R_1 H_2) \cos(I_1 H_2), I_3 = 2 \cosh(R_1 H_2) \sin(I_1 H_2), D_{12} = kccos(\sigma H_2),$$

$$D_{13} = kcsin(\sigma H_2), D_{14} = -\mu_2 \sigma sin(\sigma H_2), D_{15} = -\mu_2 \sigma cos(\sigma H_2), D_{16} = \mu_2 \sigma,$$

$$D_{17} = (\mu_3 a_2 + \eta k^2 a_2 - \eta a_2^3), D_{18} = (\mu_3 b_2 + \eta k^2 b_2 - \eta b_2^3), D_{19} = (k^2 - a_2^2),$$

$$D_{20} = (k^2 - b_2^2), T_{10} = D_{14}D_{16}D_{20} - D_{14}D_{16}D_{19} - D_{15}D_{17}D_{20} + D_{15}D_{18}D_{19},$$

$$T_{11} = -D_{12}D_{16}D_{20} + D_{12}D_{16}D_{19} - D_{13}D_{17}D_{20} + D_{13}D_{18}D_{19}$$

4. Numerical results and discussion

For the discussion of obtained results through graphical representations, we are considering the following data for the material parameters:

- (i). For viscous liquid layer [5], $\mu_1 = 1.5 \text{ NS/m}^2$, $\rho_1 = 1261.3 \text{ kg/m}^3$
- (ii). For isotropic elastic layer [1], $\mu_2 = 2.5 \times 10^{10} \text{ N/m}^2$, $\rho_2 = 2320 \text{ kg/m}^3$
- (iii). For couple stress substrate [12], $\mu_3 = 1.987 \times 10^{10} \text{ N/m}^2$, $\rho_3 = 4705 \text{ kg/m}^3$

Figures 2 and 3, depict the impacts of ratio of widths (H_2/H_1) of homogeneous isotropic layer and viscous liquid layer on the propagation of Love-type waves. It can be observed that with the increase in width ratio, both phase and damping velocities are increasing. Hence width ratio (H_2/H_1) favors both phase and damping velocities. Figures 4 and 5 present the effects of viscosity parameter (μ_1) of liquid layer on phase and damping velocities, it is observed that phase and damping velocities are increasing with increase in the value of viscosity parameter (μ_1) of viscous liquid layer. Impact of density of viscous liquid layer on the behavior of Love type waves is shown in figures 6 and 7, it is noticed that density (ρ_1) of liquid layer adversely affect phase and damping velocities, as these velocities are decreasing with the increase in the value of density parameter (ρ_1) of viscous liquid layer. The role of characteristic length parameter on the propagation behavior of Love type waves is depicted in figures 8 and 9. It is found that characteristic length parameter (l) of microstructural substrate is not favoring phase and damping velocities and these velocities are decreasing with the increase in characteristic length parameter (l).

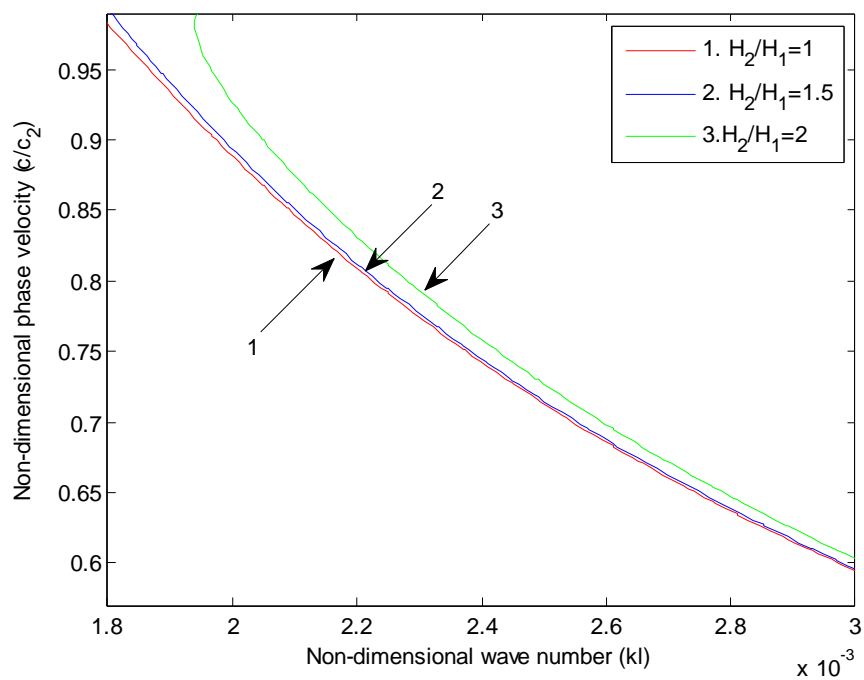


Figure 2: Profiles of phase velocity versus wave number for different values of width ratio

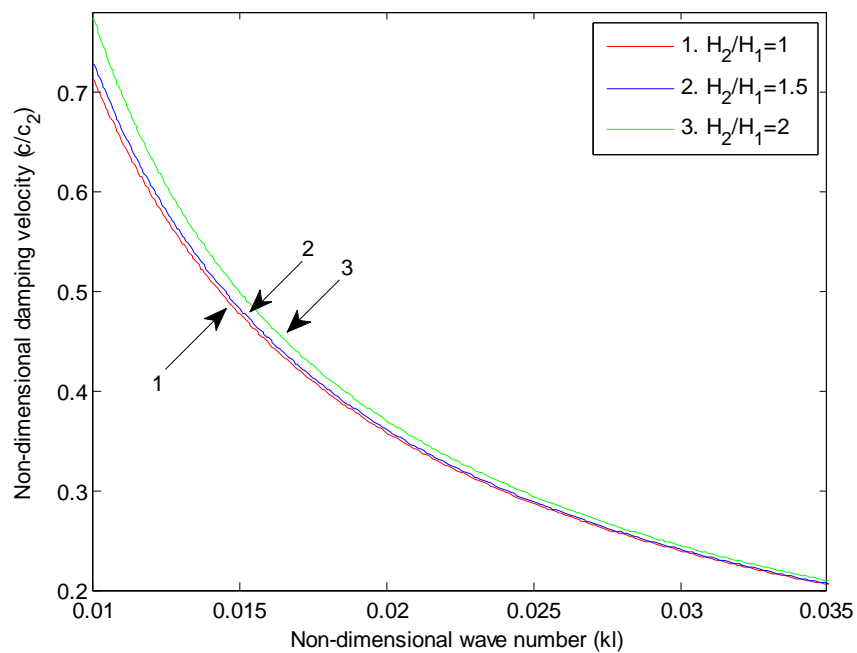


Figure 3: Profiles of damping velocity versus wave number for different values of width ratio

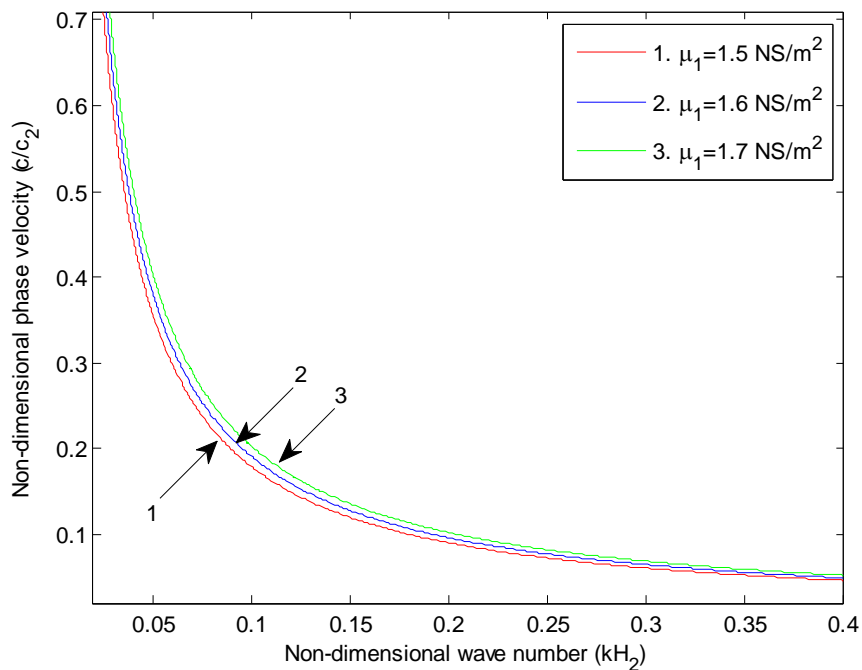


Figure 4: Profiles of phase velocity versus wave number for different values of viscosity parameter

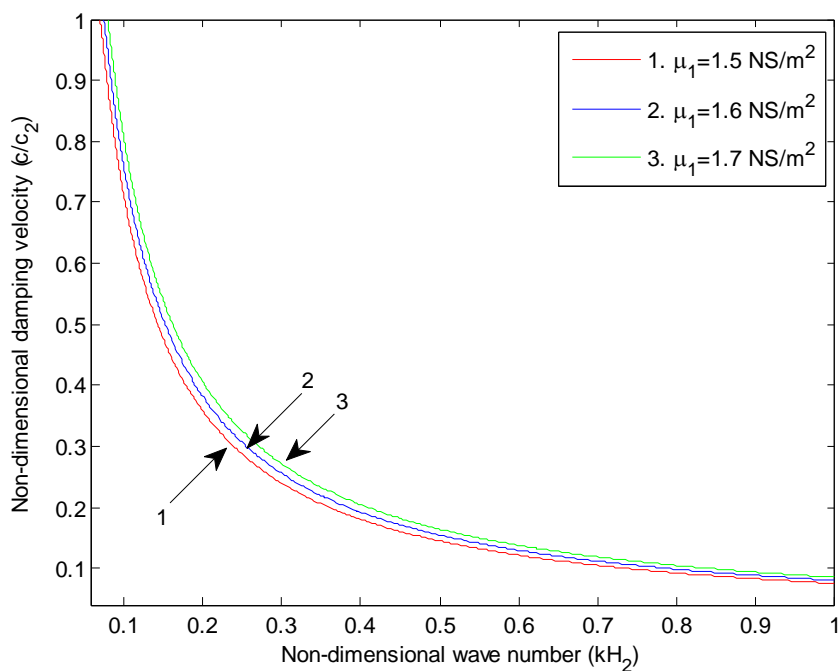


Figure 5: Profiles of damping velocity versus wave number for different values of viscosity parameter

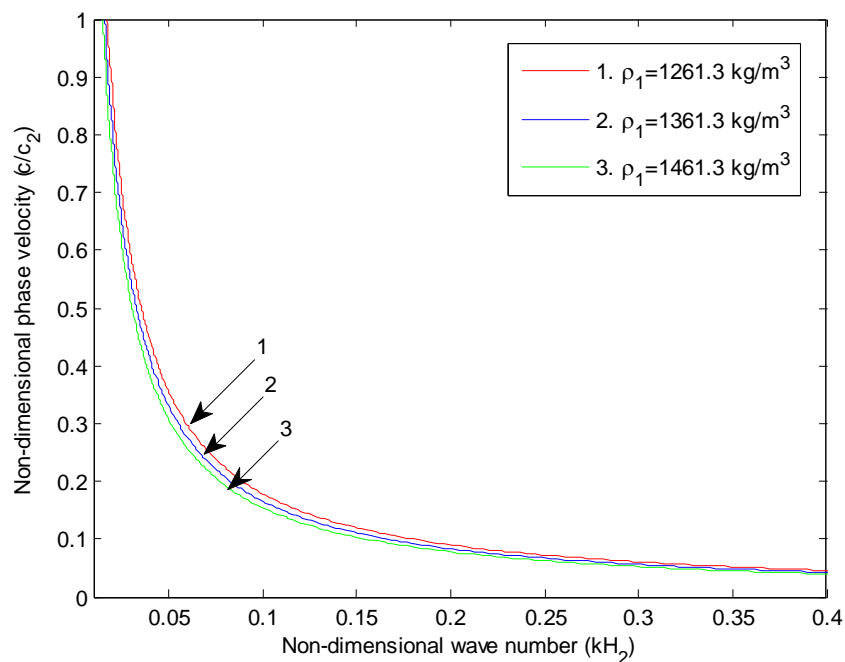


Figure 6: Profiles of phase velocity versus wave number for different values of density of liquid layer

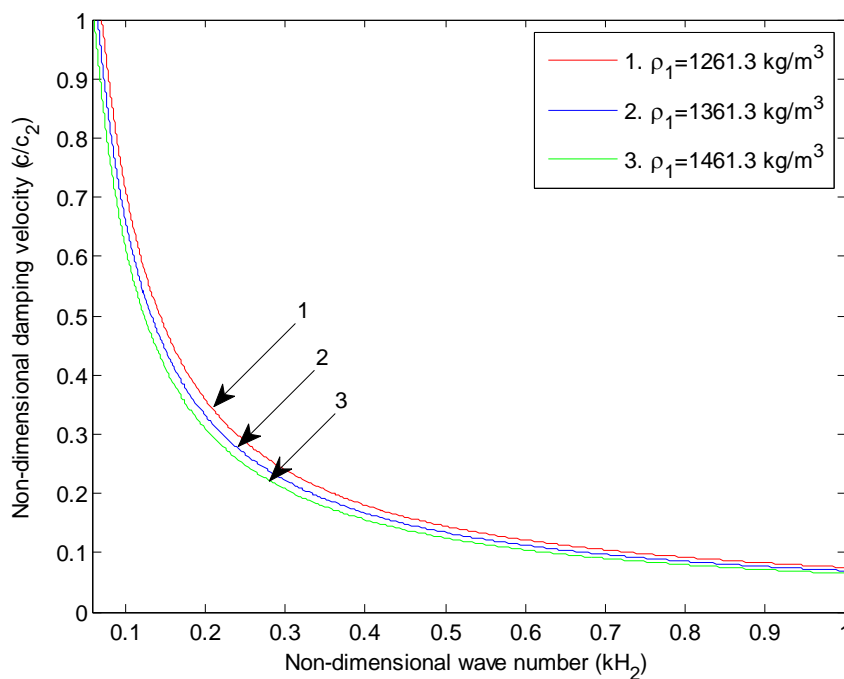


Figure 7: Profiles of damping velocity versus wave number for different values of density of liquid layer

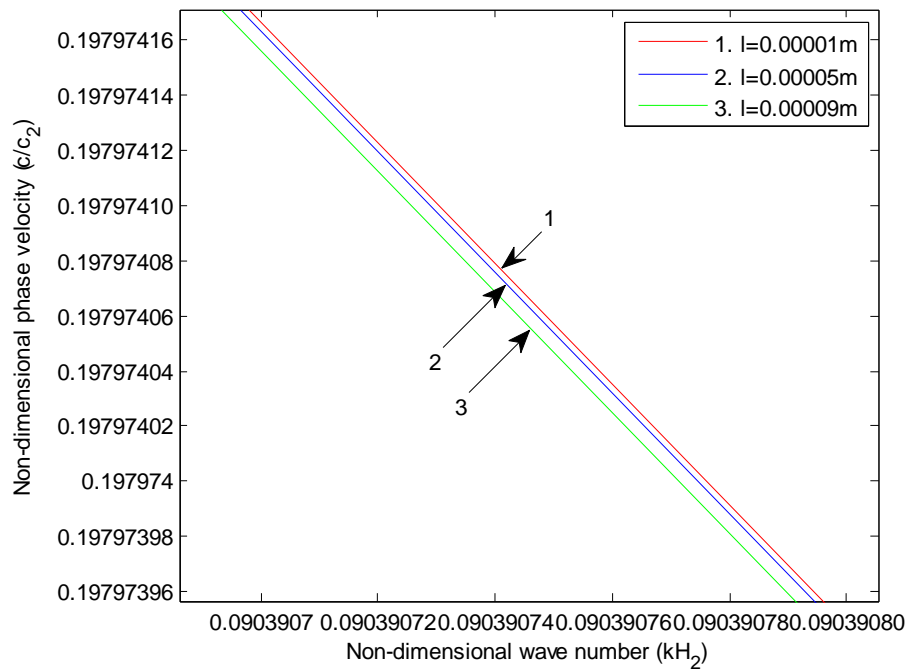


Figure 8: Profiles of phase velocity versus wave number for different values of characteristic length parameter

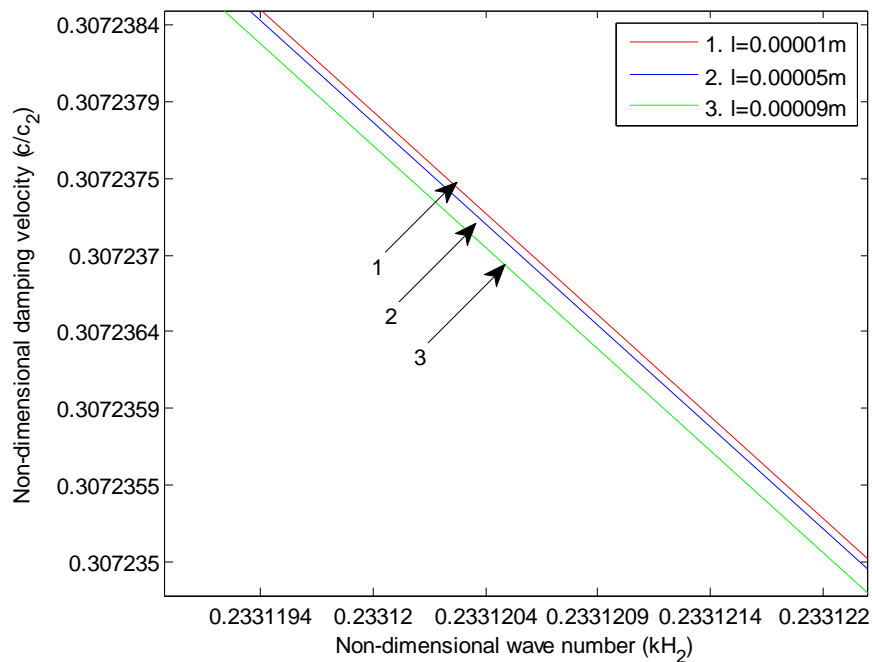


Figure 9: Profiles of damping velocity versus wave number for different values of characteristic length parameter

5. Conclusion

Following main observations are found through present analysis:

- It is found that with the increase in width ratio (H_2/H_1), both phase and damping velocities are increasing and width ratio goes in the favor of phase and damping velocities.
- Phase and damping velocities are increasing with the increasing value of viscosity parameter (μ_1) of viscous liquid layer. It is further noticed that, density parameter (ρ_1) of liquid layer adversely affect phase and damping velocities, as these velocities are decreasing with the increase in the value of density parameter (ρ_1) of viscous liquid layer.
- Characteristic length parameter (l) of microstructural substrate goes against phase and damping velocities and these velocities are decreasing with the increase in characteristic length parameter (l).

6. References

- [1] A.K. Singh, Z. Parween, S. Kumar and A. Chattopadhyay, "Propagation characteristics of transverse surface wave in a heterogeneous, layer clad with a piezoelectric stratum and an isotropic substrate", *J intel. Mat. Syst. Str.*, vol. 24(4), pp. 636-652, Aug. 2017.
- [2] A.R Hadjesfandiari, G.F. Dargush, "Couple stress theory for solids", *Int. J solids struct.*, vol. 48, pp. 2496-2510, Sept. 2011.
- [3] G. Nie, J. Liu, Y. Kong, and X. Fang, "SH Waves in $(1 - x)\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$ - $x\text{PbTiO}_3$ Piezoelectric Layered Structures Loaded with Viscous Liquid", *Acta Mech. Solida Sin.*, vol. 29(5), pp. 479-489, Oct. 2016.
- [4] G.T. Piliposian, A.S. Avetisyan and K.B. Ghazaryan, "Shear wave propagation in periodic phononic/photonic piezoelectric medium", *Wave Motion*, vol. 49, pp. 125-134, Jan. 2012.
- [5] J. Baroi, S.A. Sahu and M.K. Singh, "Dispersion of polarized shear waves in viscous liquid over a porous piezoelectric substrate", *J intel. Mat. Syst. Str.*, vol. 29(9), pp. 2040-2048, Mar. 2018.

- [6] J. Cui, J. Du, and J. Wang, "Effects of viscous liquid on SH wave propagation in layered viscoelastic/piezoelectric structure", Proceedings of the IEEE international ultrasonics symposium, Chicago, IL, pp. 1971-1974, 2014.
- [7] Q. Ma, J. Jiao, P. Hu, X. Zhong, B. Wu, C. He, "Excitation and detection of shear horizontal waves with electromagnetic acoustic transducers for nondestructive testing of plates", Chin. J Mech. Engg. vol. 27(2), pp. 428-436, Mar. 2014.
- [8] R. Kakar, "Dispersion of Love in an isotropic layer sandwiched between orthotropic and prestressed inhomogeneous half space", Lat. Am. J. solids struct., vol. 12(10), pp. 1934-1949, May 2015.
- [9] S.T. Ten, U. Hashim, S.C.B. Gopinath, W.W. Liu, K.L. Foo, S.T. Sam, S.F.A. Rahman, C.H. Voon, A.N. Nordin, "Highly sensitive escherichia coli shear horizontal surface acoustic wave biosensor with silicon dioxide nanostructures", Biosens. Bioelectron., vol. 93, pp. 146–154, July 2017.
- [10] V. Sharma, S. Kumar, "Dispersion of Rayleigh waves in a microstructural couple stress substrate loaded with liquid layer under the effects of gravity", Arch. Acoust. Vol. 43(1), pp. 11–20, 2018.
- [11] V. Sharma, S. Kumar, "Effects of liquid loadings on Lamb waves in the context of size dependent couple stress theory", J Theor. App. Mech., vol.53(4), pp. 925-934, 2015.
- [12] V. Sharma, S. Kumar, "Influence of microstructure, heterogeneity and internal friction on SH waves propagation in a viscoelastic", Struct. Eng. Mech., vol. 57(4), pp. 703-716, Feb. 2016.