

Self-Focusing of Hermite-Cosh-Gaussian Chirped Pulse Laser With Density Ramp In Thermal Quantum Magnetoplasma

Vishal Thakur*

¹Department of Physics, Lovely Professional University,

G.T. Road, Phagwara 144411, India

*E-mail: vishal20india@yahoo.co.in

Abstract

In the present manuscript, we propose a theoretical model which defines propagation of Hermite-cosh-Gaussian laser pulse in thermal quantum plasma with density ramp. Further, differential equation for the beam width parameter is set up by taking expression for dielectric function and following WKB approximation. Comparison is done for the self-focusing with density ramp profile and without density ramp is reported. The former is more prominent in achieving stronger self-focusing of laser pulse as compared to the later one.

Keywords: Hermite-Cosh-Gaussian beam, density ramp, chirped laser pulse

I. INTRODUCTION

In last few years, interaction of the laser with plasma has been world widely studied because of its unique applications which include inertial confinement fusion [1-3], electron acceleration by laser [4-6], self-focusing and the harmonic generation [7-13] etc. Among these nonlinear effects one of the most important effects is self-focusing. When a high power laser beam propagates through the plasma, dielectric function of the plasma is modified due to the oscillatory velocity of electrons. The focusing and defocusing of first six TEM_{0p} Hermite-cosh-Gaussian laser in collisionless plasma was studied by Takale et al. [14].

Importance of density ramp and decentered parameter for efficient self-focusing laser was successfully reported by Kant *et al.* [15]. Also, it has been observed that, under the influence of magnetic field and plasma density ramp, the self-focusing becomes stronger up to great extent [16].

Further, in quantum plasma self-focusing of a cosh-Gaussian laser was presented by Habibi *et al.*[17], by following higher order paraxial theory. They got better results for self-focusing of the cosh-Gaussian laser beams in comparison to Gaussian beams by selecting suitable decentered parameter.

The manuscript is defined as follows: section II devoted to equations for nonlinear dielectric function and characteristics of the beam width parameter with normalized propagation distance. In section III presents results and the discussions. Final conclusion of the investigation is presented in section IV.

II. THEORETICAL CONSIDERATIONS

In cold quantum magnetoplasma can be written as

$$E(r, z) = \frac{E_0}{f(z)} \left[H_m \left(\frac{\sqrt{2}r}{r_0 f(z)} \right) \right] e^{\frac{b^2}{4}} \left\{ e^{-\left(\frac{r}{r_0 f(z)} + \frac{b}{2}\right)^2} + e^{-\left(\frac{r}{r_0 f(z)} - \frac{b}{2}\right)^2} \right\} \tag{1}$$

where H_m is the Hermite polynomial of m^{th} order, $f(z)$ represents dimensionless beam width parameters. Oscillatory velocity related to electrons is $v = eE/m_0\omega\gamma$. Here ω, e and m_0 are the angular frequency of the incident laser beam, electronic charge and rest mass respectively. Also $\gamma = \sqrt{1 + \alpha EE^*}$, represents the relativistic factor where $\alpha = e^2/m_0^2\omega^2 c^2$. The dielectric function for the nonlinear medium can be written as

$$\epsilon = \epsilon_0 + \phi(EE^*), \tag{2}$$

where $\epsilon_0 = 1 - \omega_p^2 / \omega_1^2, \omega_p^2 = 4\pi n(\xi)e^2 / m$.

$$\phi(EE^*) = \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z / c))^2 (1 - \omega_c / \omega_0)\gamma} \exp\left(\frac{\xi}{d}\right) \left[1 - \frac{\delta}{\gamma} \right]^{-1} \left[1 - \exp\left(-\frac{3m_0\gamma}{4M} \alpha EE^*\right) \right], \tag{3}$$

where $\alpha = e^2 M / 6m_0^2 \gamma^2 \omega^2 k_B T, \delta = 4\pi^2 h^2 / m^2 \omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z / c))^2 \lambda^4,$ and

$$\gamma = \sqrt{1 + e^2 EE^* / c^2 m_0^2 \omega_0^2}.$$

Further, wave equation is given as

$$\frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial^2 \vec{E}}{\partial r^2} + \frac{1}{r} \frac{\partial \vec{E}}{\partial r} + \frac{\epsilon \omega^2}{c^2} \vec{E} = 0 \tag{4}$$

Solution of the Eq. (4) is as follows

$$\vec{E} = A(r, z) \exp[i(\omega t - kz)] \tag{5}$$

where $A(r, z)$ represents complex amplitude of the electric field. Putting the expressions for \vec{E} and $A(r, z)$ in eq. (4),

$$\begin{aligned} & -2 \frac{\partial S}{\partial z} + \frac{S \omega_0^2 \exp(z/dR_d)}{c^2 k^2 dR_d} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \\ & + \frac{z \omega_0^2 \exp(z/dR_d)}{c^2 k^2 dR_d} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \frac{\partial S}{\partial z} \\ & - \frac{z S \omega^4 \exp(z/dR_d)}{2c^4 k^4 d^2 R_d^2} \left(\frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \right)^2 \\ & + \frac{z \omega_0^2 \exp(z/dR_d)}{c^2 k^2 dR_d} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \\ & - \frac{z^2 \omega_0^4 \exp(z/dR_d)}{4c^4 k^4 d^2 R_d^2} \left(\frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \right)^2 \\ & - \left(\frac{\partial S}{\partial r} \right)^2 + \frac{1}{2A_0^2 k^2} \frac{\partial^2 A_0^2}{\partial r^2} - \frac{1}{4A_0^4 k^2} \left(\frac{\partial A_0^2}{\partial r} \right)^2 \\ & + \frac{1}{2rA_0^2 k^2} \left(\frac{\partial A_0^2}{\partial r} \right) + \frac{\phi(A_0^2)}{\epsilon_0} = 0 \end{aligned}$$

(6)

and

$$\begin{aligned}
 & -\frac{1}{A_0^2} \left(\frac{\partial A_0^2}{\partial z} \right) + \frac{\omega_0^2 \exp(z/dR_d)}{c^2 k^2 dR_d} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \\
 & + \frac{z \omega_0^2 \exp(2z/dR_d)}{c^2 k^2 d^2 R_d^2} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \\
 & + \frac{z \omega_0^4 \exp(z/dR_d)}{4c^4 k^4 d^2 R_d^2} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \\
 & - \frac{z \omega_0^2 \exp(z/dR_d)}{2c^2 k^2 dR_d A_0^2} \frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \frac{\partial A_0^2}{\partial z} - \frac{\partial^2 S}{\partial r^2} \\
 & - \frac{1}{A_0^2} \frac{\partial S}{\partial r} \frac{\partial A_0^2}{\partial r} - \frac{1}{r} \frac{\partial S}{\partial r} = 0
 \end{aligned}$$

(7)

The solutions of eqs. (6) and (7) are of form

$$A_0^2 = \frac{E_0^2}{f^2(z)} \left[H_m \left(\frac{\sqrt{2}r}{r_0 f} \right) \right]^2 e^{\frac{b^2}{2}} \left\{ e^{-2\left(\frac{r}{r_0 f(z)} + \frac{b}{2}\right)^2} + e^{-2\left(\frac{r}{r_0 f(z)} - \frac{b}{2}\right)^2} + 2e^{-\left(\frac{2r^2}{r_0^2 f^2(z)} + \frac{b^2}{2}\right)} \right\} \tag{8}$$

and

$$S = \frac{r^2}{2} \beta(z) + \varphi(z) \tag{9}$$

Substituting these values in Eq. (6), we get the equations,

For m = 0

$$\begin{aligned}
 & \left[\frac{\xi \exp(\xi/d)}{2d \left\{ 1 - \frac{\omega_{p0}^2 \exp(\xi/d)}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \right\}} \left(\frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 \gamma} \right) - 1 \right] \frac{d^2 f}{d\xi^2} \\
 & - \frac{\xi \exp(\xi/d)}{2d \left\{ 1 - \frac{\omega_{p0}^2 \exp(\xi/d)}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 (1 - \omega_c / \omega_0) \gamma} \right\}} \left(\frac{\omega_{p0}^2}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2 \gamma} \right) \frac{1}{f} \left(\frac{df}{d\xi} \right)^2 \\
 & + \frac{(4 - 4b^2)}{f^3} \\
 & - \frac{4\alpha E_0^2}{f^3} \left(\frac{\omega_{p0}^2 \exp(\xi/d)}{\omega_0^2 (1 + \Delta(\omega_0 t - \omega_0 z/c))^2} \right) \left(\frac{\omega_0 r_0}{c} \right)^2 \left(1 + \frac{4\alpha E_0^2}{f^2} \right)^{\frac{3}{2}} e^{\frac{b^2}{2}} = 0
 \end{aligned} \tag{10}$$

For $m = 1$

$$\left[\frac{\xi \exp(\xi/d)}{2d \left\{ 1 - \frac{\omega_{P0}^2 \exp(\xi/d)}{\gamma \omega^2 (1 - \omega_c/\omega)} \right\}} \left(\frac{\omega_{P0}^2}{\gamma \omega^2} \right) - 1 \right] \frac{d^2 f}{d\xi^2} - \frac{\xi \exp(\xi/d)}{2d \left\{ 1 - \frac{\omega_{P0}^2 \exp(\xi/d)}{\gamma \omega^2 (1 - \omega_c/\omega)} \right\}} \left(\frac{\omega_{P0}^2}{\gamma \omega^2} \right) \frac{1}{f} \left(\frac{df}{d\xi} \right)^2 + \frac{(4 - 4b^2)}{f^3} - \frac{8\alpha E_0^2}{f^3} \left(\frac{\omega_{P0}^2 \exp(\xi/d)}{\omega^2} \right) \left(\frac{\omega r_0}{c} \right)^2 e^{\frac{b^2}{2}} (b^2 - 2) = 0 \tag{11}$$

For $m = 2$

$$\left[\frac{\xi \exp(\xi/d)}{2d \left\{ 1 - \frac{\omega_{P0}^2 \exp(\xi/d)}{\gamma \omega^2 (1 - \omega_c/\omega)} \right\}} \left(\frac{\omega_{P0}^2}{\gamma \omega^2} \right) - 1 \right] \frac{d^2 f}{d\xi^2} - \frac{\xi \exp(\xi/d)}{2d \left\{ 1 - \frac{\omega_{P0}^2 \exp(\xi/d)}{\gamma \omega^2 [(1 - \omega_c/\omega)]} \right\}} \left(\frac{\omega_{P0}^2}{\gamma \omega^2} \right) \frac{1}{f} \left(\frac{df}{d\xi} \right)^2 + \frac{(4 - 4b^2)}{f^3} - \frac{16\alpha E_0^2}{f^3} \left(\frac{\omega_{P0}^2 \exp(\xi/d)}{\omega^2} \right) \left(\frac{\omega r_0}{c} \right)^2 \left(1 + \frac{16\alpha E_0^2}{f^2} \right)^{3/2} e^{b^2/2} (5 - 2b^2) = 0 \tag{12}$$

III. RESULTS AND DISCUSSION

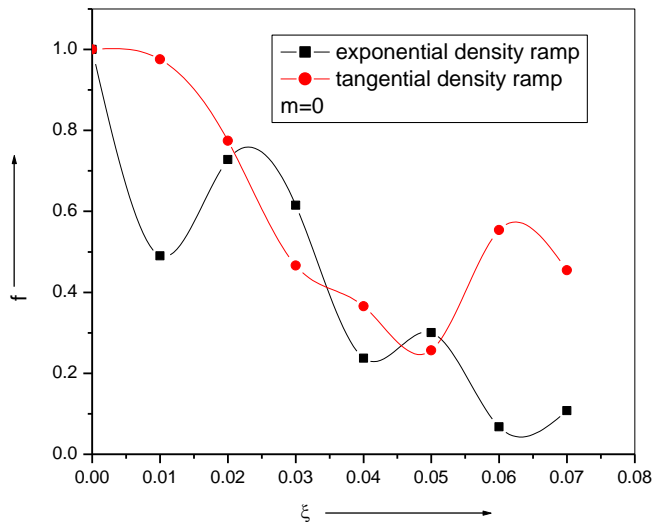
Consider frequency and the spot size of initial laser $\omega = 1.778 \times 10^{15}$ rad/s . From figure 1 (a), one may observe effect of exponential density ramp in comparison to tangential on self-focusing through magnetoplasma with relativistic effects. In the similar way for $m = 1$, figure 1 (b), noticed that, the beam passing through plasma with exponential as well as tangential density transition gets diffracted. For $m = 2$, in figure 1 (c), the strong self-focusing with early effects is noticed. Previously, Kant *et al.*[18] have observed self-focusing with ponderomotive nonlinearity under density transition and

found strong self-focusing nearly at $\xi = 0.5$. Here in this work, stronger self-focusing is noticed even at lower values of the normalized distance of propagation.

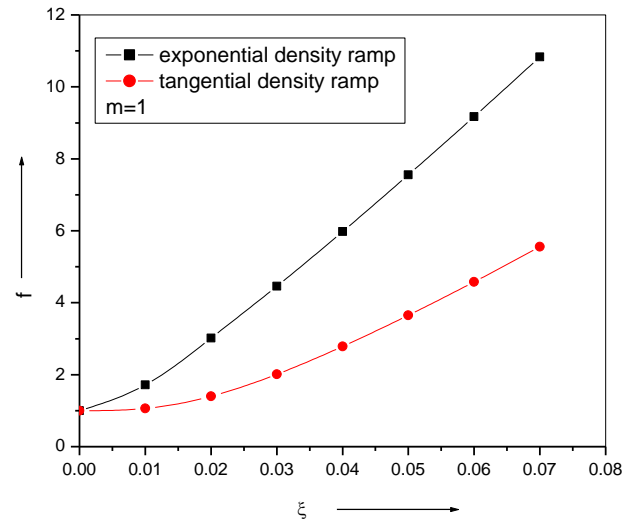
Figure 2 reveals the dependence of f on ξ for various decentered parameters b at mode indices $m = 0, 1$ and 2 . One may clearly observe that, as values of the decentered parameter enhances, self-focusing is increased and is shifted towards the smaller values of distance of propagation. Strong self-focusing of Hermite-cosh-Gaussian laser beams in collisionless magnetoplasma was studied by Patil *et al.*[19] which supports our results.

IV. CONCLUSION

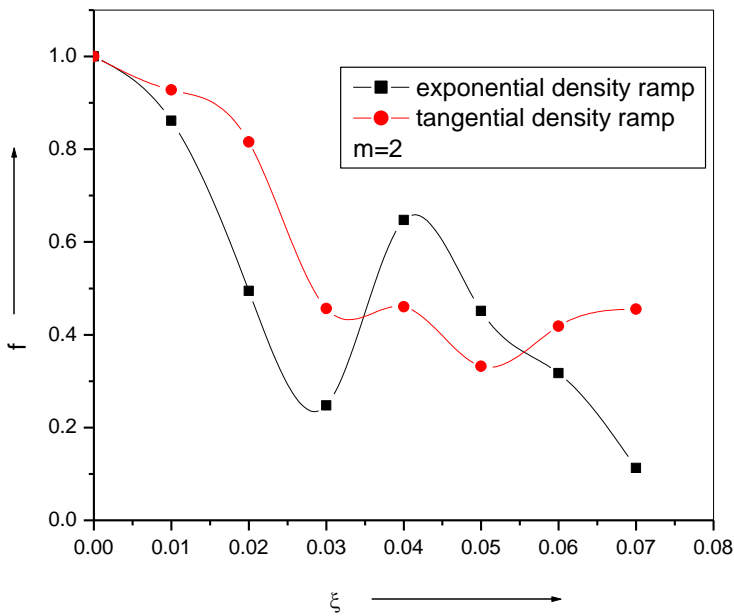
Present manuscript describes relativistic effect of self-focusing on Hermite-cosh-Gaussian laser in plasma with density transition. Using WKB approximation, enhancement in the self-focusing of Hermite-cosh-Gaussian beam is observed. It is noticed that with the density transition the spot size of Hermite-cosh-Gaussian beam reduces as it travels inside plasma. Present work may be utilized in analysis of quantum dots and laser induced fusion etc.



(a)

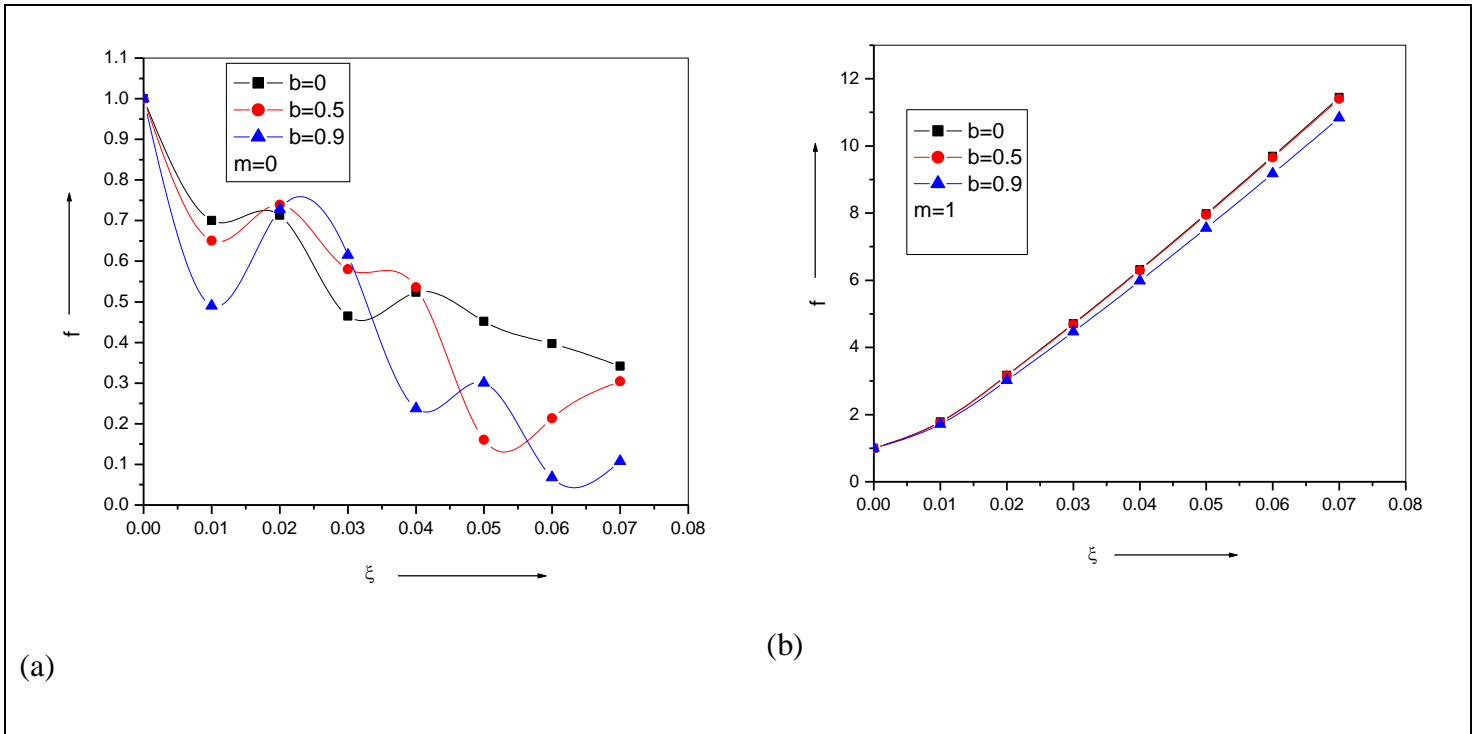


(b)



(c)

FIG. 1. Comparison between effects of exponential and tangential plasma density ramp on dependence of f on ξ for $m=0$, (a), $m=1$, (b) and $m=2$, (c). Other terms are taken as $\alpha E_0^2 = 0.1$, $\omega_{p0} / \omega = 0.75$, $\omega_c / \omega = 0.8$, $b = 0.9$, $\omega r_0 / c = 470$.



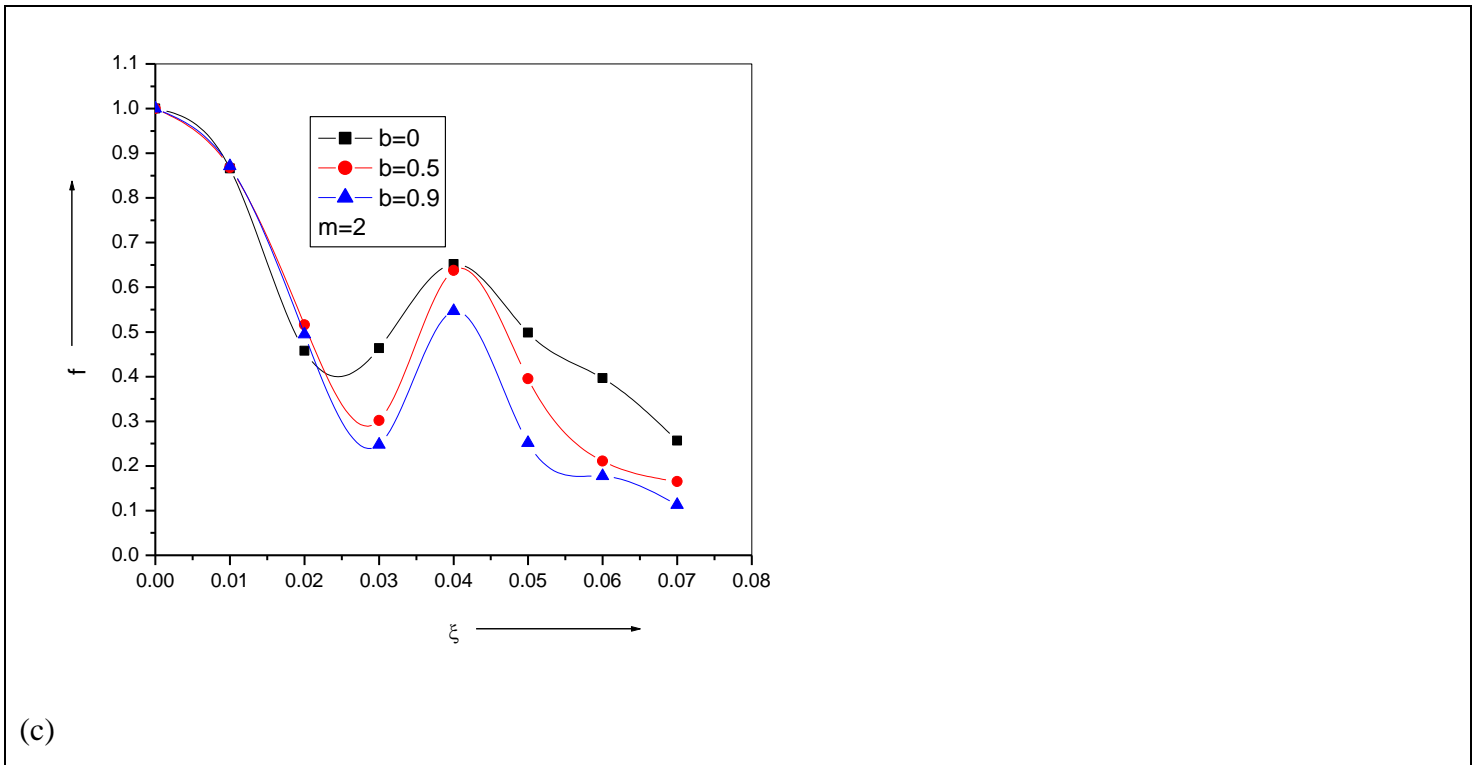


FIG. 2. Dependence of f on ξ at various decentered parameter values for $m = 0$, (a), $m = 1$, (b) and $m = 2$, (c). Other parameters are same as considered in fig. 1.

REFERENCES

[1] P. Mulser and D. Bauer, “Fast ignition of fusion pellets with superintense lasers: Concepts, problems, and perspectives,” *Laser and Particle beams*, 22, 5-12 2004.

[2] H. Hora, “New aspects for fusion energy using inertial confinement,” *Laser and Particle Beams*, 25, 37-45 2007.

[3] F. Winterberg, “Lasers for inertial confinement fusion driven by high explosives,” *Laser and Particle Beams*, 26, 127-135 2008.

[4] H. Y. Niu, X. T. He, B. Qiao and C. T. Zhou, “Resonant acceleration of electrons by intense circularly polarized Gaussian laser pulses,” *Laser and Particle Beams*, 26, 51-60 2008.

[5] J. X. Li, W. P. Zang, Y. D. Li and J. G. Tian, “Acceleration of electrons by a tightly focused intense laser beam,” *Optics Express*, 17, 11850-11859 2009.

- [6] S. Lourenco, N. Kowarsch, W. Scheid and P. X. Wang, "Acceleration of electrons and electromagnetic fields of highly intense laser pulses," *Laser and Particle Beams*, 28, 195-201 2010.
- [7] N. Kant and V. Thakur, "Second harmonic generation by a chirped laser pulse in magnetized-plasma," *Optik-International Journal for Light and Electron Optics*, 127, 4167-4172 2016.
- [8] V. Thakur and N. Kant, "Stronger self-focusing of a chirped pulse laser with exponential density ramp profile in cold quantum magnetoplasma," *Optik*, 172, 191-196 2018.
- [9] V. Thakur and N. Kant, "Optimization of wiggler wave number for density transition based second harmonic generation in laser plasma interaction," *Optik*, 142, 455-462 2017.
- [10] V. Thakur, S. Vij, V. Sharma and N. Kant, "Influence of exponential density ramp on second harmonic generation by a short pulse laser in magnetized plasma," *Optik*, 171, 523-528 2018.
- [11] V. Thakur and N. Kant, "Resonant second harmonic generation in plasma under exponential density ramp profile," *Optik*, 168, 159-164 2018.
- [12] V. Sharma, V. Thakur and N. Kant, "Third harmonic generation of a relativistic self-focusing laser in plasma in the presence of wiggler magnetic field," *High Energy Density Phys.* 32, 51-55 2019.
- [13] V. Thakur, M. A. Wani and N. Kant, "Relativistic self-focusing of Hermite-cosine-Gaussian laser beam in collisionless plasma with exponential density transition," *Commun. Theor. Phys.* 71, 736-740 2019.
- [14] M. V. Takale, S. T. Navare, S. D. Patil, V. J. Fulari and M. B. Dongare, "Self-focusing and defocusing of TEM_{0p} Hermite-Gaussian laser beams in collisionless plasma," *Optics Communications*, 282, 3157-3162 2009.
- [15] V. Nanda and N. Kant, "Enhanced relativistic self-focusing of Hermite-cosh-Gaussian laser beam in plasma under density transition," *Physics of Plasmas*, 21, 042101 2014.
- [16] V. Nanda, N. Kant and M. A. Wani, "Self-focusing of a Hermite-cosh Gaussian laser beam in a magnetoplasma with ramp density profile," *Physics of Plasmas*, 20, 113109 2013.

- [17] M. Habibi and F. Ghamari, "Improved focusing of a cosh-Gaussian laser beam in quantum plasma: higher order paraxial theory," *IEEE Transactions on Plasma Science*, 43, 2160-2165 2015.
- [18] N. Kant, S. Saralch and H. Singh, H. "Ponderomotive self-focusing of a short laser pulse under a plasma density ramp," *Nukleonika*, 56, 149-153 2011.
- [19] S. D. Patil, M. V. Takale, S. T. Navare, and M. B. Dongare, "Focusing of Hermite-cosh-Gaussian laser beams in collisionless magnetoplasma," *Laser and Particle Beams*, 28, 343-349 2010.