ISSN: 0971-1260
Vol-22-Issue-17-September-2019

On Common Fixed Point Theoremsin Complex Valued Metric Spaces

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Abstract: In the present manuscript, we establish some theorems using (E.A.)-property & (CLR)-property for two pairs of weakly compatible mappings in the framework of complex valued metric spaces.

Keywords:(E.A)-property, (CLR)-property, weakly compatible mappings, complex valued metric spaces.

1. Introduction

The famous result known as Banach's contraction principle is, if(X,d) is a complete metric space& $T:X\rightarrow X$ is a mapping satisfying $d(Tx,Ty)\leq kd(x,y)\forall x,y\in X,$ where k is a nonnegative numbers with $k<1,$ then a mapping T has a unique fixed point in X. This famous principle is the foundation stone of nonlinear analysis. The theory has immense applications not only in pure mathematics, but also hasgained a remarkable scope in applied mathematics, economics, mechanics, physics, engineering and other sciences. Fixed point and common fixed point of mappings has been obtained by the researcher using various definitions. [see [1-28] and the references cited therein). In the year 2011, Azam et al.[1] introduced a more generalized space called complex valued metric space. Later, number of results has been given by researchers in the framework of complex valued metric space. The below mentioned definitionsAzam et al.[1]are required in the sequel.

Take$\mathbb{C}$as a set of complex numbers&let$z_{1,}$ $z_2\in\mathbb{C}$. Consider a partial order '$\preccurlyeq$'on C as below:

$z_1\preccurlyeq z_2$ iff$Re\,(z_1)\leq Re(z_2),\ Im(z_1)\leq Im(z_2)$.

From this, it follows that $z_1\preccurlyeq z_2$ if one of the below mentioned conditions is satisfied:

(a) $Re\,(z_1)=Re\,(z_2)\,,\ Im(z_1)<Im(z_2)$.
(b) $Re\,(z_1)<Re\,(z_2),\ Im(z_1)=Im(z_2)$.
(c) $Re\,(z_1)<Re\,(z_2),\ Im(z_1)<Im(z_2)$.
(d) $Re\,(z_1)=Re\,(z_2)\,,\ Im(z_1)=Im(z_2)$.

In particular,$z_1\preccurlyeq z_2$ if (a) or (b)or (c) is satisfied and$z_1\prec z_2$ if only (c) is satisfied.

Note: The following statements hold:

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(a) $a, b \in \mathbb{R} \& a \leq b \implies az \leq bz, \forall z \in \mathbb{C}$;
(b) $0 \leq z_1 \nleq z_2 \implies |z_1| < |z_2|$;
(c) $z_1 \leq z_2 \& z_2 < z_3 \implies z_1 < z_2$.

Definition 1.1.Let a non-empty set be X&$d: X \times X \to \mathbb{C}$ satisfies:

(a) $0 \preccurlyeq d(x,y) \forall x, y \in X$ and $\mathrm{d}(x,y) = 0$ iff $x = y$;
(b) $d(x,y) = d(y.x) \forall x, y \in X$;
(c) $d(x,y) \preccurlyeq d(x,z) + d(z,y) \; \forall x, y, z \in X$.

Then, d is s. t. ba complex valued metric defined on X and (X, d) is s. t. b a complex valued metric space.

A point $x \in X$ is s.t.ban interior point of $D \subseteq X$ if there is $0 \prec r \in \mathbb{C}$ s.t $B(x,r) = \{y \in X: d(x,y) < r\} \subseteq D$. D, a subset of X is s.t.b open if each point of D is an interior point of D.

A point $x \in X$ is s. t. b a limit point of Dif for every $0 \prec r \in \mathbb{C}$, $B(x,r) \cap (D \setminus X) \neq \phi$. D, a subset of X is s. t. b closed if each limit point of D belongs to D.

Consider $\{x_n\}$ in X and $x \in X$. If $\forall c \in \mathbb{C}$, with $0 \prec c$, there is $n_0 \in N$ such that $\forall n > n_0, d(x_n, x) \prec c$, then x is s. t. ba limit of $\{x_n\}$ and we represent is as $\lim_{n\to\infty} x_n = x$ *or xn→x as n→∞.*

If $\forall c \in \mathbb{C}$, with $0 \prec c$, there is $n_0 \in N$ such that for every $n > n_0, d(x_n, x_{n+m}) \prec c$, then $\{x_n\}$ is s.t.b a Cauchy sequence in $(X, d)$and (X, d) is s. t. b a complete complex valued metric space if every Cauchy sequence is convergent in (X, d)..

Lemma 1.2 ([1]) Consider as complex valued metric space (X, d) and let $\{x_n\}$ be a sequence in X then

(a) $\{x_n\}$ converges to x iff $|d(x_n, x)| \to 0$ as $n \to \infty$.
(b) $\{x_n\}$is a Cauchy sequence iff$|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.

Definition 1.3 ([2]) A pair of self mappings $S, A: X \to X$ is weakly compatible if there is a point $v \in X$ such that $Av = Sv$, then $ASv = SAv$ for each $u \in X$

Definition 1.4([28]) Now, define the 'max' (maximum) function for$' \preccurlyeq '$ the partial order relation by:

(a) $\max\{z_1, z_2\} = z_2 \iff z_1 \leq z_2$.
(b) $z_1 \leq \max\{z_2, z_3\} \implies z_1 \leq z_2, or\ z_1 \leq z_3$
(c) $\max\{z_1, z_2\} = z_2 \iff z_1 \leq z_2 \ or\ |z_1| \leq |z_2|$

This definition results in the following lemmas

Lemma 1.5([28]) Consider $z_1, z_2, z_3, \dots\dots \in C$ and partial order relation $\preccurlyeq$ is defined on $\mathbb{C}$. Then following results can be proved easily:

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(a) If $z_1 \leq \max\{z_2, z_3\}$then$z_1 \leq z_2$ if $z_3 \leq z_2$;
(b) If $z_1 \leq \max\{z_2, z_3, z_4\}$then$z_1 \leq z_2$ if $\max\{z_3, z_4\} \leq z_2$;
(c) $z_1 \leq \max\{z_2, z_3, z_4, z_5\}$then$z_1 \leq z_2$ if $\max\{z_3, z_4, z_5\} \leq z_2$,and so on.

Definition1.6([28]) Let (X, d) a complex valued metric space and A & $S$are two maps from $X$to X.Then

(a) the pair$(A, S)$is said to satisfy (E.A.)- property,if there exists $\{x_n\}$in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t, \text{for some } t \in X.$$

(b) A and S are said to satisfy the (CLR) common limit range in the range of S property, if there exists $\{x_n\}$in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = St, \text{for some } t \in X.$$

2. Main Result

This section contains some results on common fixed points using (E.A.)-property & (CLR)-property.

Theorem 2.1: Let (X, d)a complex valued metric space and $A, B, S, T: X \to X$ be four self-mappingsatisfying:

$i)$ $A(X) \subseteq T(X), B(X) \subseteq S(X)$;

$$ii) \forall\, x, y \in X \;\&\; 0 < k < 1,$$

$$d(Ax, By) \preccurlyeq k\, Max\left\{d(Ax, Sx), d(By, Ty), d(Sx, Ty), \frac{1}{2}\left(d(Ax, Ty) + d(By, Sx)\right)\right\};$$

iii) $(A, S)$and$(B, T)$are weakly compatible pairs;

$iv)$ either(A, S)or (B, T) satisfy (E.A.)-property.

If the range of $S(X)$or $T(X)$ is a complete subspace of X, then A, B, S and T have a unique common fixed point in X.

Proof: First of all, suppose that (B, T) satisfy (E.A.)-property. Then,there is$\{x_n\}$ in Xsuch that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = t \text{for some } t \in X. \qquad \dots (2.1)$$

Further,$B(X) \subseteq S(X)$, therefore there is $\{y_n\}$ in X such that$Bx_n = Sy_n$.
Hence, $\lim_{n\to\infty} Sy_n = t$. We claim that $\lim_{n\to\infty} Ay_n = t$.

If not, then put$x = y_n$, $y = x_n$ in (ii), we obtain

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$$d(Ay_n, Bx_n) \preccurlyeq kMax\left\{d(Ay_n, Sy_n), d(Bx_n, Tx_n), d(Sy_n, Tx_n), \frac{1}{2}\big(d(Ay_n, Tx_n) + d(Bx_n, Sy_n)\big)\right\}$$

Taking$n \to \infty$ and using (2.1), we have

$$d(Ay_n, t) \preccurlyeq kMax\left\{d(Ay_n, t), 0,0, \frac{1}{2}(d(Ay_n, t) + 0)\right\}$$

Then $|d(Ay_n, t)| \leq k\left|Max\left\{d(Ay_n, t), 0,0, \frac{1}{2}d(Ay_n, t)\right\}\right|$

$|d(Ay_n, t)| \leq k|d(Ay_n, t)| < |d(Ay_n, t)|$as $0 < k < 1$,

a contradiction. Hence, $\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = t$.

Now, let $S(X)$be a closed subspace of X,therefore$t = Su$ for some u ∈ X.

Thus,

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = t = Su. \qquad \dots (2.2)$$

We claim $Au = Su$. Put $x = u$ and y $= x_n$ in (ii), we obtain

$$d(Au, Bx_n) \preccurlyeq kMax\left\{d(Au, Su), d(Bx_n, Tx_n), d(Su, Tx_n), \frac{1}{2}\big(d(Au, Tx_n) + d(Bx_n, Su)\big)\right\}$$

Taking $n \to \infty$ and using (2.2), we have

$$d(Au, t) \preccurlyeq kMax\left\{d(Au, t), d(t, t), d(t, t), \frac{1}{2}\big(d(Au, t) + d(t, t)\big)\right\}$$

$$= kMax\left\{d(Au, t), 0,0, \frac{1}{2}d(Au, t)\right\}$$

$d(Au, t) \preccurlyeq kd(Au, t)$

Then $|d(Au, t)| \leq |d(Au, t)| < |d(Au, t)|$ $as\ \ 0 < k < 1$, a contradiction.Thus, u is a coincidence point of (A, S).

Now,$(A, S)$is weakly compatibility, this implies $ASu = SAu$ or At = St.

On the other side, $A(X) \subseteq T(X)$, there is v in X such that Au = Tv.

Hence,$Au = Su = Tv = t$. Now, we prove that $Bv = Tv = t$.

Put= u , $y = v$ in (ii), we have

$$d(Au, Bv) \preccurlyeq k\ Max\left\{d(Au, Su), d(Bv, Tv), d(Su, Tv), \frac{1}{2}\big(d(Au, Tv) + d(Bv, Su)\big)\right\}$$

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or$d(t, Bv) \preccurlyeq k\ Max\left\{d(t,t), d(Bv,t), d(t,t), \frac{1}{2}\left(d(t,t) + d(Bv,t)\right)\right\}$

or$d(t, Bv) \preccurlyeq k\ Max\left\{0, d(Bv,t), 0, \frac{1}{2} d(Bv,t)\right\}$

or$|d(t, Bv)| \leq k|d(Bv,t)| < |d(Bv,t)|\ as\ \ 0 < k < 1$, a contradiction. Thus $Bv = t$.
Hence $Bv = Tv = t$.

Further, $(B, T)$are weakly compatible, this implies that $BTv = TBv, or\ Bt = Tt$.

Thus,t is a common coincidence point of A, B, S and T.

Next, to prove 't' is a common fixed point. Put $x = u$ and $\mathrm{y} = \mathrm{t}$ in (ii), we obtain

$$d(t\ , Bt) = d(Au, Bt \preccurlyeq k\ Max\left\{d(Au,Su), d(Bt,Tt), d(Su,Tt), \frac{1}{2}\left(d(Au,Tt) + d(Bt,Su)\right)\right\}$$

$$= k\ Max\left\{0,0, d(t,Bt), \frac{1}{2}\left(d(t,Bt) + d(Bt,t)\right)\right\}$$

$$= k\ Max\{0,0, d(t,Bt), d(t,Bt)\},$$

or $|d(t,Bt)| \leq k|\ Max\{0,0,d(t,Bt),d(t,Bt)\}| \leq k|d(t,Bt)| < |d(t,Bt)|$, a contradiction.

Hence, $Bt = t$.Thus$At = Bt = St = Tt = t$.

Similar reasons arise if we take$T(X)$a complete subspace of X. On the same lines,using (E.A.)-propertyfor (A,S), we get a similar result.

Uniqueness,let $w \neq t$ be another common fixed point ofA, B, S and T in X.Then,put$x = w, y = t\ in\ (ii)$, we have

$$d(Aw, Bt) \preccurlyeq k\ Max\left\{d(Aw,Sw), d(Bt,Tt), d(Sw,Tt), \frac{1}{2}\left(d(Aw,Tt) + d(Bt,Sw)\right)\right\}$$

$$= k\ Max\left\{d(w,w), d(t,t), d(w,t), \frac{1}{2}\left(d(w,t) + d(t,w)\right)\right\}$$

$$= k\ Max\{0,0, d(w,t), d(w,t)\}$$

$|d(w,t)| \leq k|d(w,t)| < |d(w,t)|$, a contradiction. Thus $w = t$.This implies uniqueness.

Theorem2.2: Let(X, d) a complex valued metric space and $A, B, S, T: X \to X$ be four self-mapping satisfying:

$i)\ A(X) \subseteq T(X), B(X) \subseteq S(X)$;

$$ii) \forall\ x, y \in X\ \&\ \ 0 < k < 1,$$

$$d(Ax, By) \preccurlyeq k\ Max\left\{d(Ax,Sx), d(By,Ty), d(Sx,Ty), \frac{1}{2}\left(d(Ax,Ty) + d(By,Sx)\right)\right\};$$

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iii) $(A,S)$and$(B,T)$are weakly compatible pairs.

If(A,S) satisfy (CLR_A) property or (B,T) satisfy (CLR_B) property,then A,B,S and T have a unique fixed point in X.

Proof: Firstly, suppose that (B,T) satisfy (CLR_B) property. Thus, there is a sequence

$\{x_n\}$in$X$ such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = Bx \text{for some } x \in X. \quad \ldots(2.3)$$

Further,$BX \subseteq SX$, we have $Bx = Su$, for some $u \in X$.We claim that $Au = Su = t$ (say).
Put $x = u$ and $y = x_n$ in (ii),we obtain

$$d(Au, Bx_n) \preccurlyeq k\ Max\left\{d(Au,Su), d(Bx_n,Tx_n), d(Su,Tx_n), \frac{1}{2}\left(d(Au,Tx_n) + d(Bx_n,Su)\right)\right\}$$

Taking $n \to \infty$ and using (2.3), we have

$$d(Au, Bx) \preccurlyeq k\ Max\left\{d(Au,Su), d(Bx,Bx), d(Su,Bx), \frac{1}{2}\left(d(Au,Bx) + d(Bx,Su)\right)\right\}$$

$$\preccurlyeq k\ Max\left\{d(Au,Bx), d(Bx,Bx), d(Bx,Bx), \frac{1}{2}\left(d(Au,Bx) + d(Bx,Bx)\right)\right\}$$

$$\preccurlyeq k\ Max\left\{d(Au,Bx), 0, 0, \frac{1}{2} d(Au,Bx)\right\}$$

$$|d(Au,Bx)| \leq k\left|Max\left\{d(Au,Bx), 0, 0, \frac{1}{2} d(Au,Bx)\right\}\right|$$

$|d(Au,Bx)| \leq k|d(Au,Bx)| < |d(Au,Bx)|as0 < k < 1$, a contradiction Thus $Au = Su$.

Hence $Au = Su = Bx = t$.

Now,(A,S) are weakly compatible, this implies that $ASu = SAu$ or $At = St$.

If, $A(X) \subseteq T(X)$, therefore thereis some $v \in X$ such that$Au = Tv$. Thus $Au = Su = Tv = t$.

Now, we show that $Bv = Tv = t$. For this, put $x = u, y = v$ in (ii), we obtain

$$d(Au, Bv) \preccurlyeq k\ Max\left\{d(Au,Su), d(Bv,Tv), d(Su,Tv), \frac{1}{2}\left(d(Au,Tv) + d(Bv,Su)\right)\right\}$$

or$d(t,Bv) \preccurlyeq k\ Max\left\{d(t,t), d(Bv,t), d(t,t), \frac{1}{2}\left(d(t,t) + d(Bv,t)\right)\right\}$

or$d(t,Bv) \preccurlyeq k\ Max\left\{0, d(Bv,t), 0, \frac{1}{2} d(Bv,t)\right\}$

or$|d(t,Bv)| \leq k\left|Max\left\{0, d(Bv,t), 0, \frac{1}{2} d(Bv,t)\right\}\right|$

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or$|d(t,Bv)| \leq k\ |d(Bv,t)| < |d(Bv,t)|$ as $0 < k < 1$, a contradiction.Hence, $Bv = t$. Thus, $Bv = Tv = t$.

Further, $(B,T)$are weakly compatible, this implies $BTv = TBv\ or\ Bt = Tt$.Thus, a common coincidence point of A, B, S and T is t.

Now, to prove that 't' is a common fixed point.Substitute $x = u$ and $\mathrm{y} = \mathrm{t}$ in (ii), weobtain

$$d(t,Bt) = d(Au,Bt)$$

$$\preccurlyeq k\ Max\left\{d(Au,Su), d(Bt,Tt), d(Su,Tt), \frac{1}{2}\left(d(Au,Tt) + d(Bt,Su)\right)\right\}$$

$$= k\ Max\left\{d(t,t), d(Bt,Bt), d(t,Bt), \frac{1}{2}\left(d(t,Bt) + d(Bt,t)\right)\right\}$$

$$= k\ Max\{0,0, d(t,Bt), d(t,Bt)\}.$$

Hence, $|d(t,Bt)|\ \leq k|d(t,Bt)| < |d(t,Bt)|$ as $0 < k < 1$, a contradiction.Thus,$Bt = t$.Hence, $At = Bt = St = Tt = t$.We can prove uniquenesseasily.

On the similar way,if(A,S)satisfy(CLR_A)-property, we will obtain the unique commonfixed point of A,B,S and T.

Remark2.3 : In the present manuscript, we claim the existence and uniqueness of common fixed point using (E.A.)–property and (CLR)-property. In (E.A.)–property, we need the condition of closedness of subspace. But, in case of (CLR)-property no such condition is required. Hence, (CLR)-property is an interesting tool to check the existence and uniqueness of common fixed point.

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