

Fundamental Approach and Applications of Laplace Transform

ReetuMalhotra^a, Gittaly^b

^a*Chitkara University Institute of Engineering and Technology, Chitakara University, Punjab, Punjab, India*

reetu.malhotra@chitkarauniversity.edu.in

^b*School of Electronics and Electrical Engineering, Lovely Professional University, Phagwara, Punjab, India*
gittaly.25304@lpu.co.in

Abstract-

An integral transmute that is useful in resolving complex linear differential equations. It discovers extensive applications in numerous areas of physics, economics, control engineering, optics, mathematics, pharmacy, electrical engineering, signal processing and optometry. Laplace transform construed as a conversion of time domain functions to the frequency domain functions. In complex angular frequency, input and output behaves as input. In this study we have discussed various Laplace transform in technical fields like Mechanical systems, Engineering, Electronics, image processing and also in solving complex differential equations.

Keywords-Laplace transform, inverse Laplace transform, Applications

1. Introduction

By times many mathematicians studied on transforms and gave their own transform, there are numerous transforms with numerous applications today. Transform are used to convert one function into other function. Transforms are not only restricted to engineering and applied sciences but they have a large application other than engineering for example, transform is used in communication system for signal processing, used in televisions theatre screens for image processing. Transforms can be visualized as a filter which is used to differentiate mixed frequency signals into different frequency wave signal. There many types of transform but mathematicians have analyzed integral transform due to their large applications. The first integral transform was given by Abel [1] for spherically symmetrical and axially symmetrical functions. Another English mathematician Bateman [2] gave Bateman transform which is used for the transformation of holomorphic functions in three dimensions. Fourier [3] gave his transform which is used to transform functions which are continuous and discontinuous i.e. functions are converted into sine and cosine periodic series. This transform has the significant role in the history of mathematics. Great French mathematician, Laplace invent transform named as Laplace transform.

Laplace [4] gave Laplace transform which has large application in solving differential equations of LCR (inductance, capacitance and resistance) which is important part of electronics. Out of the integral transform the Laplace transform plays a significant role in almost every field. Actually, a Laplace transform time domain function into frequency domain without altering the function values. Researchers use Laplace transform to transform calculations having compound differential equations to simple and quadratic equations which are easily solvable. Laplace transform use derivative to transform into a numerous domain variable. So, for the n th order equations, Laplace transform convert equations into n th degree polynomial values according to the fixed rules set. It is a well-known mathematical procedure for resolving differential equations. Various real-world applications are solved using Laplace transform in an easiest way, because the conversion of one domain to alternative domain is easier to solve.

In a two-way communication scenario, Laplace transformation is conducted, as information is directed over intermediate medium i.e. radio sets, FM/AM stereos [5] and cellular phones. For transmitting information, it is first transformed into time domain and then overlaid on medium and finally information spreads. In the receiving end, again time functions are transformed into frequency domain. The Laplace equation for a function $f(t)$ can be given as:

$$L = \int_0^{\infty} e^{-st} f(t) dt \quad (1)$$

1.1 Laplace transform properties

Laplace Transform exhibits various properties as and used in various real time applications as

(a) Linearity:

$$L[af(t) + bf(t)] = aF(s) + bF(s) \quad (2)$$

(b) Derivative of Laplace transform:

$$L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^+) \quad (3)$$

(c) N th order derivative

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1}f(0^+) \quad (4)$$

(d) Integral

$$L\left[\int_0^t f(\tau) d\tau\right] = \frac{F(s)}{s} \quad (5)$$

These all properties have significant role in for analysing various technical and non-technical fields.

2. Applications of Transform in Different fields:

Laplace transform found useful in almost all disciplines for several applications described as follows.

a) System Modelling:

In system modelling, Laplace transform is useful for to shorten calculations [6], in an easier way, where lengthy differential calculations are used. Where, the inverse Laplace transform can be evaluated by employing various properties of Laplace transform. Using the inverse of Laplace transform we calculate every parameter of the model such as its temperature, pressure, volume etc. by solving the Laplace with application of partial differential equations.

b)Electrical Circuits Analysis

Laplace transform is used in electrical circuits for the study of linear time-invariant systems. As shown in Figure 1. Laplace transform is used in RLC circuit to calculate the current and voltage across each component of the circuit.

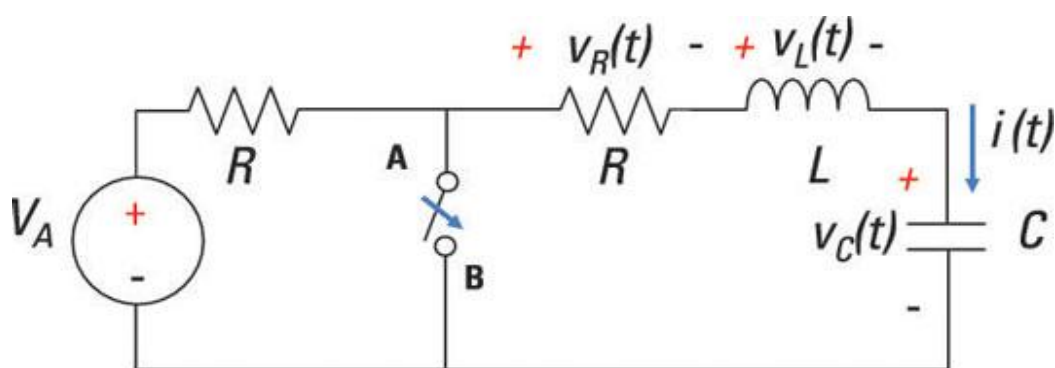


Figure 1: Circuit analysis using Laplace Transform

We applied KVL equation in the circuit and final element equation is represent as

$$v_R(t) + v_L(t) + v(t) = 0$$

(6)

Where Ohm's law defines, voltage across the resistor where, $i(t) = i_L(t)$ and circuit is associated in series, Laplace transforms of $I(s) = I_L(s)$ and represent as

$$v_R(t) = i(t)R$$

(7)

Using inductors equation:

$$v_L(t) = L \frac{di_L(t)}{dt}$$

(8)

Capacitor equation can be written as:

$$v_C(t) = \frac{1}{C} \int_0^t i(\tau) d\tau + v_C(0)$$

(9)

By combining equations (i),(ii) and (iii)

$$L \frac{di_L(t)}{dt} + i_L(t)R + \frac{1}{C} \int_0^t i(\tau) d\tau + v_C = 0$$

(10)

c) *Electronic Circuits Analysis*

Electronics engineers widely used Laplace transform for solving and explaining differential equations for the analysis and study of various electronic circuits. In integrated circuits, get output from devices, Laplace transform is used.

d) *Digital Signal Processing*

DSP based difficulties can be solved without using various transform [7]. Laplace transform analyses, altered variables functions and generate desired results. In heat experimentation

study, Laplace transform is used to measure the extent of changes in desired output corresponds to input w.r.t temperature. This is a well-organized and easier method to regulate processes that are guided by differential equations.

e) Nuclear Physics

For radioactive decay, to achieve the true form Laplace transform is used and reviewing the possible analytic segment of Nuclear Physics. Where n_1 and n_2 are number of nuclei and λ_1 and λ_2 are wavelengths of nuclei energies.

$$n_2(t) \approx \frac{n_1(0)\lambda_1}{\lambda_2} [1 - e^{-\lambda_2 t}] \quad (11)$$

f) Process Controls

It helps analyse the variables, which when altered, produces desired manipulations in the result. For example, while study heat experiments, transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an efficient and easier way to control processes that are based on differential equations.

g) Inverse Laplace transform applications

To evaluate inverse Laplace transform, of specified variables or function, identify the specified function either in the given comparable form or divided the function into various expression using partial fractions [8].

h) Differential equations

Oliver Heaviside [8] evolving a technique for the efficient explanation of simple differential equations with constant coefficients values using initial conditions. Without concluding the arbitrary constant values, differential equations can be solved in an easy way by using differential equations.

i) Vibrating Mechanical Systems

For exploratory the suspension structure of car, the significant system elements considered are mass (measured in kg), springs (assessed in Nm^{-1}) and damper (assessed in Nsm^{-1}). Dampers and springs are used to attach suspension links of the car with body.

Spring, mass and damper structure can model using Hooke's and Newton's law. Therefore, differential equations are used to represent system and these equations solve can solve by Laplace Transform [9]

j) Application in economics

For accounting Laplace transform application in economics, investment projects can be considered. Present value of cash receipts payments and transactions are given by

$$[Present Value]_t = \sum_{t=1}^T \frac{Cashvalue(t)}{(1 + rate\ of\ discount)^t} \quad (12)$$

This represents the cash flows of present value of a stream of and after taking Laplace transform, it can be considered as

$$[PV]_r = \int_0^{\infty} e^{-rate\ of\ discount} cashvalue(t) dt \quad (13)$$

3. Conclusion

Laplace Transform is a tool to abridge complex problems in the simplify manner. This paper represents the Laplace transform applications in different fields, like Electronics, control process, physics, control process and mechanical process etc. It provides great flexibility in several applications. With the affluence of Laplace transforms applications in countless technical applications, lots of software have realized to simulate Laplace equations directly.

References

[1] F.K. Yin, W. Y. Han, and J. Q. Song, "Modified Laplace decomposition method for Lane-Emden Type differential equations," International Journal of Applied Physics and Mathematics, vol.3, no. 2, pp. 98-102, 2013.

- [2] M.A. Mohamed and M. S. Torchy, "Numerical solution of nonlinear system of partial differential equations by the Laplace decomposition method and the Padé approximation," *American Journal of Computational Mathematics*, vol. 3, no.3, pp.1-3, 2013.
- [3] E. Yusufoglu, "Numerical Solution of duffing equation by the Laplace decomposition algorithm", *Applied Mathematics and Computation*, vol. 177, no.2, pp. 572-580, 2006.
- [4] S. Kazem, "Exact Solution of Some Linear Fractional Differential Equations by Laplace Transform", *International Journal of Nonlinear Science*, vol.16, no.1, pp.5-6, 2013.
- [5] L. Kexue and P. Jigen, "Laplace transform and fractional differential equations," *Applied Mathematics Letters*, vol.24, pp.2019–2023.2011.
- [6] S. Gupta, D. Kumar and J. Singh, "Numerical study for systems of fractional differential equations via Laplace transform," *Journal of the Egyptian Mathematical Society*, pp.6-7, 2015
- [7] V Banfi, *The origin of the solar system according to P S Laplace (Italian)*, *Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur.* 112, 1978.
- [8] F Barone, *The epistemology of Pierre-Simon de Laplace (Italian)*, *Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur.* 112, 1978, suppl., 7.
- [9] L Zabell, Buffon, Price, and Laplace : scientific attribution in the 18th century, *Arch. Hist. Exact Sci.* vol. 39, no. 2, 1988.