

A modelling on stream selection of undergraduate students in education sector under fuzzy TOPSIS analysis

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ABSTRACT

In modern education system, students are suffering from choosing proper stream in undergraduate courses. Due to high availability of choices, there are a significant number of streams for a specific course provided to a student. In this respect, this paper aims to develop a mathematical model for the decision making in choosing a particular stream out of several alternatives. In this research, several alternatives are given to a student and each alternative corresponds to some criterion. The student must choose an alternative under the basis of the educational background of the student. All alternatives, criterion, and weights are assigned with fuzzy membership, and non-membership function. The entire method of choosing the correct alternative is created under intuitionistic fuzzy TOPSIS method. The model is validated with proper numerical experimentation.

Keywords: *Intuitionistic fuzzy set, MCDM, TOPSIS, stream selection.*

1 INTRODUCTION

Multi-Criteria Decision Method (MCDM) is a sub-disciplinary branch of operational research related to the development of mathematical and analytical methods to enable a single decision-maker or team to subjectively analyze a number of alternatives under several number of performance criteria.[1]. There are several MCDM methods to solve real- world problems but “the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method” is used most satisfactorily to solve decision problems in different application areas.[2]. Firstly Ching-Lai Hwang and Yoon [3] has developed “The Technique for Order of Preference by Similarity to Ideal Solution” (TOPSIS) in 1981, followed by Yoon in 1987 [4] and Hwang, Lai and Liu in 1993 [5].

Now-a-days there are lot of advancements in the field of education. Every Educational institute is trying to provide number of different disciplines having variety of new streams and pathways to the new joining students. When student wants to join new institute after completion of his/her secondary education, he/she may face a lot of confusion in choosing appropriate stream for higher study. Intuitionistic fuzzy TOPSIS method can provide students a proper way to choose correct stream based upon career preference.

Boren et al.[6] have used Intuitionistic fuzzy TOPSIS approach to test renewable energy technologies for the generation of electricity in Turkey. Chu and Su [7] used the TOPSIS method to choose a fixed “seismic shelter for urban evacuation”. A "fixed seismic evacuation shelter" is a location where migrants stay longer and provide intended support. Zhongyou [8] suggested a most suitable and effective procedure to select foreign players in CBA games using TOPSIS method. Chen [9] applied the TOPSIS approach to the fuzzy world of decision-making problems. The author expressed the criterion in "Triangular Fuzzy Numbers (TFNs)" and he proposed the method of linear normalization and the vertex method to determine the distance measurements for the final results. Tlig and Rebai [10] have

developed an intuitionistic Fuzzy TOPSIS method to solve MCDM problem of comparing the service quality of five airports in North Africa. Jahanshaloo et al.[11] have done the extension of “the TOPSIS method” to solve MCDM problems having Fuzzy data. He used "Triangular Fuzzy Numbers (TFNs)” for the ranking of each alternative and the importance of each criterion. Alpha-cuts are used to normalize the fuzzy numbers. The TOPSIS approach has also been extended by Abo-Sinna and Amer [12] to address "multi-objective, nonlinear programming problems" on a large scale.

2 MATHEMATICAL MODELS

2.1 NOTATION USED

$\mu_S(x)$ membership function

$\nu_S(x)$ non membership function

$\pi_S(x)$ hesitation function

A intuitionistic fuzzy decision matrix

M set of grades of importance

A' intuitionistic fuzzy weighted decision matrix

P^+ intuitionistic fuzzy positive ideal solution

P^- intuitionistic fuzzy negative ideal solution

S_i^+, S_i^- measures of separation

CC_i relative closeness coefficient

2.2 Assumption

1. Different students have given their order of preference of pathways depending upon economic conditions.
2. Various institutes are giving option of multi-streams (alternatives) to the students.
3. Multi-Pathways (Criteria) are to be provided to the students.
4. All alternatives, criterion and weights are assigned with fuzzy membership and non membership function.

2.3 Intuitionistic Fuzzy set (IFS)

In this paper we are presenting a Mathematical model which is based on intuitionistic fuzzy TOPSIS method. Intuitionistic Fuzzy sets(IFS) are first introduced by Atanassov [13] to deal with the problem of vagueness and uncertainty. Lots of decision making problems [14–19] are resolved by using IFS.

Let X be any finite set then “Intuitionistic Fuzzy set (IFS)” S in X is given as

$$S = \{ \{x, \mu_S(x), \nu_S(x)\} | x \in X \}$$

where $\mu_S : X \rightarrow [0,1]$ and $\nu_S : X \rightarrow [0,1]$ are representing the membership and non-membership grade respectively of the element x and

$$0 \leq \mu_S(x) + \nu_S(x) \leq 1 \quad \forall x \in X \tag{1}$$

The function $\pi_S(x) = 1 - \mu_S(x) - \nu_S(x)$ is called the hesitation function of x to S where

$$0 \leq \pi_S(x) \leq 1 \tag{2}$$

More value of hesitation function means there is more uncertainty about x .

Let X_1 and X_2 be any two IFSs then the following property of fuzzy multiplication is given by

$$X_1 \otimes X_2 = \{ \{x, \mu_{X_1}(x) \cdot \mu_{X_2}(x), \nu_{X_1}(x) + \nu_{X_2}(x) - \nu_{X_1}(x) \cdot \nu_{X_2}(x)\} | x \in X \} \tag{3}$$

2.4 Topsis Method

TOPSIS is a useful and practical technique for ranking and selecting through distance measurement of several externally determined alternatives. This approach is based on the idea that the specified alternatives have the minimum distance from the "positive ideal solution" and the maximum distance from the "negative ideal solution"[20]. In fact, "the ideal solution" is a solution that maximizes the criteria for benefit and minimizes the criteria for cost.

The Intuitionistic fuzzy TOPSIS method can be summarized as

Step I. First, determine an "intuitionistic fuzzy decision matrix".

Suppose A is an "intuitionistic fuzzy decision matrix" whose various alternatives are represented by m rows ($i=0,1,2,3,\dots,m$) and several criterion are represented by n columns ($j=0,1,2,3,\dots,n$) then the "decision matrix" A is defined as

$$A = \begin{bmatrix} (\mu_{A_{11}}(x_1), \nu_{A_{11}}(x_1), \pi_{A_{11}}(x_1)) & (\mu_{A_{12}}(x_2), \nu_{A_{12}}(x_2), \pi_{A_{12}}(x_2)) & \dots & (\mu_{A_{1n}}(x_n), \nu_{A_{1n}}(x_n), \pi_{A_{1n}}(x_n)) \\ (\mu_{A_{21}}(x_1), \nu_{A_{21}}(x_1), \pi_{A_{21}}(x_1)) & (\mu_{A_{22}}(x_2), \nu_{A_{22}}(x_2), \pi_{A_{22}}(x_2)) & \dots & (\mu_{A_{2n}}(x_n), \nu_{A_{2n}}(x_n), \pi_{A_{2n}}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_{m1}}(x_1), \nu_{A_{m1}}(x_1), \pi_{A_{m1}}(x_1)) & (\mu_{A_{m2}}(x_2), \nu_{A_{m2}}(x_2), \pi_{A_{m2}}(x_2)) & \dots & (\mu_{A_{mn}}(x_n), \nu_{A_{mn}}(x_n), \pi_{A_{mn}}(x_n)) \end{bmatrix}$$

Step II. Define an" intuitionistic fuzzy weighted decision matrix".

Different criteria can have the different importance and the level of importance can vary for each criterion. So, we can provide a grade of importance to each criterion. Let M be the set of grades of importance given to each criterion. An IFS M in X is written as

$$M = \{x, \mu_M(x), \nu_M(x) | x \in X\} \tag{4}$$

where $\mu_M : X \rightarrow [0,1]$ and $\nu_M : X \rightarrow [0,1]$ are representing the membership and non-membership grade respectively of $M \in X$.

where
$$0 \leq \mu_M(x) + \nu_M(x) \leq 1 \quad \forall x \in X \tag{5}$$

The function $\pi_M(x) = 1 - \mu_M(x) - \nu_M(x)$ is called the hesitation function of M to X.

Let A_i and M be two IFSs of the set X then using definition given by ‘‘Atanassov’’ [13]:

$$A_i \otimes M = \left\{ \left\langle x, \mu_{A_i}(x) \cdot \mu_M(x), \nu_{A_i}(x) + \nu_M(x) - \nu_{A_i}(x) \cdot \nu_M(x) \right\rangle \mid x \in X \right\} \tag{6}$$

and

$$\pi_{A_i \cdot M}(x) = 1 - \nu_{A_i}(x) - \nu_M(x) - \mu_{A_i}(x) \cdot \mu_M(x) + \nu_{A_i}(x) \cdot \nu_M(x) \tag{7}$$

The ‘‘intuitionistic fuzzy weighted decision matrix’’ is defined as:

$$A' = \begin{bmatrix} (\mu_{A_1M}(x_1), \nu_{A_1M}(x_1), \pi_{A_1M}(x_1)) & (\mu_{A_2M}(x_2), \nu_{A_2M}(x_2), \pi_{A_2M}(x_2)) & \dots & (\mu_{A_nM}(x_n), \nu_{A_nM}(x_n), \pi_{A_nM}(x_n)) \\ (\mu_{A_1M}(x_1), \nu_{A_1M}(x_1), \pi_{A_1M}(x_1)) & (\mu_{A_2M}(x_2), \nu_{A_2M}(x_2), \pi_{A_2M}(x_2)) & \dots & (\mu_{A_nM}(x_n), \nu_{A_nM}(x_n), \pi_{A_nM}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_1M}(x_1), \nu_{A_1M}(x_1), \pi_{A_1M}(x_1)) & (\mu_{A_2M}(x_2), \nu_{A_2M}(x_2), \pi_{A_2M}(x_2)) & \dots & (\mu_{A_nM}(x_n), \nu_{A_nM}(x_n), \pi_{A_nM}(x_n)) \end{bmatrix}$$

Step III. Calculate ‘‘intuitionistic fuzzy positive ideal’’ and ‘‘intuitionistic fuzzy negative ideal’’ solutions.

Let B and C be representing be ‘‘benefit criteria’’ and ‘‘cost criteria’’ respectively. Let P^+ and P^- be an ‘‘intuitionistic fuzzy positive ideal solution’’ and ‘‘intuitionistic fuzzy negative ideal solution’’. Then

$$P^+ = (p_1^+, p_2^+, \dots, p_n^+), p_k^+ = (\mu_{A_j \cdot M}^+(x_k), \nu_{A_j \cdot M}^+(x_k), \pi_{A_j \cdot M}^+(x_k)), k = 1, 2, \dots, n \tag{8}$$

$$P^- = (p_1^-, p_2^-, \dots, p_n^-), p_k^- = (\mu_{A_j \cdot M}^-(x_k), \nu_{A_j \cdot M}^-(x_k), \pi_{A_j \cdot M}^-(x_k)), k = 1, 2, \dots, n \tag{9}$$

where

$$\mu_{A_j \cdot M}^+(x_k) = \{(\max_{ij} \{\mu_{A_j \cdot M}(x_k)\} \mid k \in B), (\min_{ij} \{\mu_{A_j \cdot M}(x_k)\} \mid k \in C)\} \tag{10}$$

$$\nu_{A_j \cdot M}^+(x_k) = \{(\min_{ij} \{\nu_{A_j \cdot M}(x_k)\} \mid k \in B), (\max_{ij} \{\nu_{A_j \cdot M}(x_k)\} \mid k \in C)\} \tag{11}$$

$$\mu_{A_j \cdot M}^-(x_k) = \{(\min_{ij} \{\mu_{A_j \cdot M}(x_k)\} \mid k \in B), (\max_{ij} \{\mu_{A_j \cdot M}(x_k)\} \mid k \in C)\} \tag{12}$$

$$v_{A_j \cdot M}^-(x_k) = \{(\max_{ij} \{v_{A_j \cdot M}(x_k)\} | k \in B), \{(\min_{ij} \{v_{A_j \cdot M}(x_k)\} | k \in C)\} \tag{13}$$

STEP IV. Determine the measures of separation using normalized Euclidean distance.

Different type of normalized distance such as Hamming distance, Euclidean distance etc. can be used to calculate measures of separation. In this paper, we used normalized Euclidean distance.

Let S_i^+ and S_i^- be the measures of separation of every alternative from “intuitionistic fuzzy positive ideal” and “intuitionistic fuzzy negative ideal” solutions. Then

$$S_i^+ = \sqrt{\frac{1}{2n} \sum_{k=1}^n [(\mu_{A_j \cdot M}(x_k) - \mu_{A_j \cdot M}^+(x_k))^2 + (v_{A_j \cdot M}(x_k) - v_{A_j \cdot M}^+(x_k))^2 + (\pi_{A_j \cdot M}(x_k) - \pi_{A_j \cdot M}^+(x_k))^2]} \tag{14}$$

$$S_i^- = \sqrt{\frac{1}{2n} \sum_{k=1}^n [(\mu_{A_j \cdot M}(x_k) - \mu_{A_j \cdot M}^-(x_k))^2 + (v_{A_j \cdot M}(x_k) - v_{A_j \cdot M}^-(x_k))^2 + (\pi_{A_j \cdot M}(x_k) - \pi_{A_j \cdot M}^-(x_k))^2]} \tag{15}$$

STEP V. Determine the “relative closeness coefficient”.

The “relative closeness coefficient” of each alternative A_i is determined by using formula

$$CC_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad \text{where } 0 \leq CC_i \leq 1 \ (0 \leq i \leq m) \tag{16}$$

STEP VI. Assign ranking to each alternative.

Ranking of each alternative can be done according to the calculate values of CC_i 's in descending order.

3. Numerical Analysis

Suppose there are three students of non-medical who want to join some engineering institute. Every student has different priorities for each pathway depending upon their economic conditions, skill and interest for choosing stream of engineering.

Suppose there are four streams(alternatives) such as “Civil, Electrical, Computer Science (CSE) and Mechanical” of engineering to be chosen which are denoted as A_1, A_2, A_3, A_4 . There are four pathways (criterion) given for every stream which are “Government Job, Corporate, Entrepreneur and Higher studies”. The three students have their own importance of each criteria. The “linguistic variables” for importance of criterion and “Intuitionistic fuzzy numbers” associated with each variable are given in table 1.

Table I:Linguistic variables for importance of criterion	
Linguistic variables	IFNs
very high	(0.86,0.10)
high	(0.78,0.09)
low	(0.5,0.45)
very low	(0.37,0.56)

The three students have given their importance to different criterion given in table II.

Table II: The importance of criterion			
Criterion	Weights		
	S1	S2	S3
Government Job	very high	high	high
Corporate Sector	high	very high	very low
Entrepreneurship	very low	low	low
Higher Education	low	very low	very high

First of all, calculate Intuitionistic fuzzy decision matrix for Student 1 given as

		Government	Corporate	Entrepreneur	Higher studies
A =	Civil	(0.75, 0.1, 0.15)	(0.5, 0.3, 0.2)	(0.2, 0.7, 0.1)	(0.60, 0.25, 0.15)
	Electrical	(0.72, 0.11, 0.19)	(0.6, 0.22, 0.18)	(0.4, 0.5, 0.1)	(0.5, 0.35, 0.15)
	CSE	(0.4, 0.41, 0.19)	(0.8, 0.13, 0.07)	(0.7, 0.1, 0.2)	(0.6, 0.15, 0.25)
	Mechanical	(0.7, 0.14, 0.16)	(0.5, 0.2, 0.3)	(0.25, 0.65, 0.1)	(0.75, 0.14, 0.11)

Now calculate Intuitionistic fuzzy weighted decision matrix given as

		Government	Corporate	Entrepreneur	Higher studies
A' =	Civil	(0.645, 0.19, 0.165)	(0.39, 0.363, 0.247)	(0.074, 0.868, 0.058)	(0.3, 0.588, 0.112)
	Electrical	(0.619, 0.199, 0.182)	(0.468, 0.29, 0.242)	(0.148, 0.78, 0.072)	(0.25, 0.643, 0.107)
	CSE	(0.344, 0.469, 0.187)	(0.624, 0.208, 0.168)	(0.259, 0.604, 0.137)	(0.3, 0.533, 0.167)
	Mechanical	(0.602, 0.226, 0.172)	(0.39, 0.272, 0.338)	(0.092, 0.846, 0.062)	(0.375, 0.527, 0.098)

In order to show how the elements of Intuitionistic fuzzy weighted decision matrix are calculated, the government criteria for civil is calculated as

$$\mu_{A_1, M}(x_1) = 0.75 * 0.86 = 0.645 \tag{17}$$

$$v_{A_1, M}(x_1) = 0.1 + 0.1 - 0.1 * 0.1 = 0.19 \tag{18}$$

$$\pi_{A_1, M}(x_1) = 1 - 0.1 - 0.1 - (0.75 * 0.86) + (0.1 * 0.1) = 0.165 \tag{19}$$

Government and corporate are benefit criterion whereas entrepreneur and higher education as cost criterion. Based on these calculated values of intuitionistic fuzzy positive ideal solution (P^+) and intuitionistic fuzzy negative ideal solution (P^-) are given as

$$P^+ = \{(0.645,0.19,0.165), (0.624,0.208,0.168), (0.074,0.868,0.058), (0.25,0.643,0.107)\} \quad (20)$$

$$P^- = \{(0.344,0.469,0.187), (0.39,0.363,0.247), (0.259,0.604,0.137), (0.375,0.527,0.098)\} \quad (21)$$

Now separation measure for each alternative is calculated by using normalized Euclidean distance formula given in table III. In order to explain how the value of separation measure is determined, “intuitionistic positive separation measure” is given for the civil branch.

$$S_1^+ = \sqrt{\frac{1}{8} \left\{ (0.645 - 0.645)^2 + (0.19 - 0.19)^2 + (0.165 - 0.165)^2 + (0.39 - 0.624)^2 + (0.363 - 0.208)^2 \right\}} \\ = 0.106 \quad (22)$$

After calculating values of S_i^+ and S_i^- for each alternative, relative closeness coefficient is calculated as given in Table III.

Table III: relative closeness coefficient of each alternative for student S₁			
STREAM	S+	S-	CC _i
Civil	0.106	0.1720	0.641819288
Electrical	0.0789	0.18994	0.685532084
Cse	0.1927	0.1092	0.499350481
Mechanical	0.1229	0.1712	0.582114927

Similarly, for other two student’s relative closeness coefficient for each alternative is calculated as given in Table IV and decision of stream to be chosen will be taken by arranging the relative closeness coefficients in descending order.

Table VI: relative closeness coefficient of each alternative for student S₂			
STREAM	CC ₁	CC ₂	CC ₃
Civil	0.641819288	0.353188879	0.463214352
Electrical	0.685532084	0.345331075	0.435918218
CSE	0.499350481	0.39724738	0.338783091
Mechanical	0.582114927	0.277736238	0.442359152

Graphical representation of choice of each student based upon pathway preferences is shown in Fig.1.

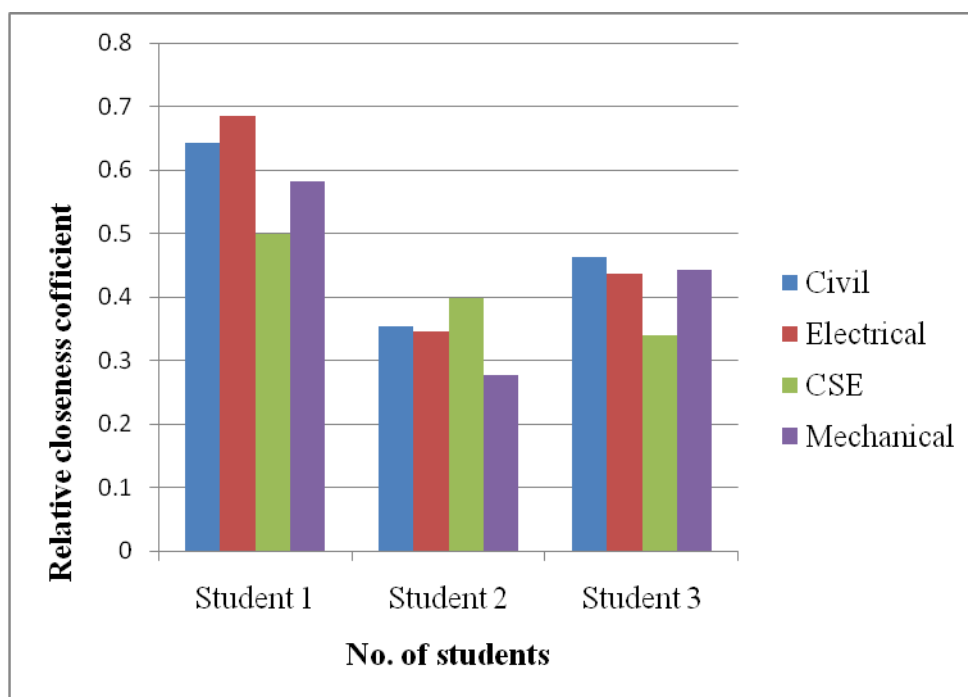


Fig.1: Order of stream preference of different students

For student S_1 , different streams have been ranked as:

Electrical>Civil>Mechanical>CSE

For student S_2 , different streams have been ranked as:

CSE>Civil>Electrical>Mechanical

and for student S_3 , different streams have been ranked as:

Civil>Mechanical>Electrical>CSE

So electrical has been ranked as best stream for student 1, Computer Science for student 2 and Civil for student 3.

4. Conclusion

In this study, the use of “intuitionistic fuzzy TOPSIS method” has been done to do the selection of engineering stream based on certain criterion. During this process, Civil, Electrical, Computer Science and Mechanical Engineering streams are considered as four alternatives. Government Job, Corporate, Entrepreneur, and Higher Studies were considered as evaluation criteria that are generally taken as the major pathways associated with each stream. Different students have their own priorities for each pathway. The “intuitionistic fuzzy TOPSIS method” is an effective method for decision-making with multi-criteria, as it can deal with a vague perception of the opinions of decision-makers. The results

of this article specify that Electrical is the best Stream for Student 1, Computer Science is best for student 2 and Civil is best for student 3.

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