

Comparative Analysis of Different Structure of the Crisp and Fuzzy Regression Equations Using Realistic Data

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Abstract:

Regression analysis is an efficient analytical method with multiple applications. The approach establishes a relationship with an approximate variance of measurement error between the dependent and independent variables. If a phenomenon under consideration not only has stochastic variance, but in some way is also ambiguous, it is more natural for the data to try a fuzzy functional relationship, which may be fuzzy or crisp. In this paper the data of RCF Kapurthala Punjab for the year 2017-2018 have been taken to obtain the relationship between the demand value and cost of Linke Hoffmann Bush (LHB) three tier AC coach train through different form of crisp and fuzzy regression equations using least square methods. The comparative analysis of these relations we easily estimated the predicted the future demand value based on the cost of the coach.

Keywords: Regression equation, fuzzy regression, least square method, demand value and cost etc. ,

1. Introduction

Regression is a very important numerical tool. There are lots of uses of regression analysis in different areas. After having the confirmed that two variables are closely related in approximating or expecting the value of one variable with respect to another variable. For example, the yield of rice and the amount of rain are related closely with one another then we identify the amount of rain required for the production of definite quantity of rice. The relation between two variables is conveyed by regression analysis and with the help of this prediction is possible.

The vocabulary meaning of the term “Regression” is the act of returning or going back. The term was first used by “sir Francis Galton in 1877 while studying the relationship between the heights of father and sons. The line describing the tendency to regress or going back was called by Galton a” Regression Line”. In order to explain the trend of a given group of points, we still used the regression line to draw a given line. The purpose of this paper is to solve a single problem using different regression methods. In this paper we are using different kind of curves to solve the regression like linear, curvilinear regression, fuzzy regression. Regression analysis is used for prediction as mentioned above, just as on the basis of the current run rate of the team we predict the number of runs in the coming overs with the help of regression, these are regression applications.

Fuzzy regression is mainly used in statistics in which we represent the dependence of one variable on another variable. We are using the empirical data to represent the Fuzzy regression. The data of coach wise material schedule for the year 2017-18 for ELEC sections and all classes for LWACCN. We also use algorithms to find out the regression. There are different kinds of motivations that we use, such as crisp parameters and fuzzy data, and fuzzy parameters and crisp data.

2. Literature Review

To solve the problem of linear regression analysis we use Fuzzy Regression. By different authors there are lots of methods to solve the fuzzy regression as mentioned below: [1] convenient for the fuzzy linear function to linear regression model and recognized methods of system demonstration for an indefinite phenomenon. The imprecision of the calculation system was symbolized by the fuzzy linear function. To evaluate the given data, the linear system used by [2-3] to find the fuzzy parameters this method is useful on the data. To enhance the precision, [4] planned a two-stage creation of a linear regression model. His determination had a major impact on the use of fuzzy regression and the practical use environment. When the data was not appropriate for statistical regression analysis, [5-7] found a practical alternative that is fuzzy regression. On the basis of contrast of observed membership function and the expected membership function, the fuzzy regression model is evaluated by [8]. Otherwise they use the indistinctness model as they use fuzzy linear regression and fuzzy linear least square

regression[9]. To solve the problem of fuzzy regression, [10] considered a fuzzy linear regression model with non-fuzzy input and output data type. He presented a preassigned k-limiting value. These values are resolute to solve the current problem. To find out the abnormal values were affected by the k-limiting values [11-14] automatic change point detection method offered to resolve the piecewise fuzzy regression. They bring together possible and necessity regression models, and function worked in different ways in different portions of the range of crisp input variables. In [15] it was suggested that functional relationship between the independent and dependent variable were in fuzzy surroundings. In many fuzzy regression models the input is crisp and the output is fuzzy. In [16], the authors introduced that the distance fuzzy least square, is effective and an alternative model to estimate the fuzzy parameters by taking fuzzy input and fuzzy output. Ali Azadeh, *et al.* [17] applied fuzzy regression model to gas consumption in Iran. They equated with conventional regression methods and fuzzy regression model to the crisp data. They detected that fuzzy algorithm has superior presentation than conventional regression analysis on the basis of fuzzy linear regression model. Xu Jialu & Lu Qiu jun [18], established this for Predicting the GDP growth using outlier effect. They recommended that the performance of fuzzy linear regression is superior to the conventional linear regression model.

3. Mathematical Preliminaries

3.1 Regression: The term which explains the relationship between two or more number of variables in the form of original unit of data that mathematical term is stated as regression. There are two kinds of variables that are used in regression:

- a) **Dependent variable or Regressed or explained variable:** whose value is predicted
- b) **Independent variable or repressor or predictor or explanatory variable:** This explanatory variable is used for the approximation.

3.2 FUZZY NUMBER: The value of a fuzzy number is approximate rather than accurate. Any fuzzy number can be of the form of a function whose domain is specified set (usually like the set of real numbers), and whose range varies from non-negative real numbers between 0 and 1000. Each numerical value in the domain is allocated a specific “grade of membership” where 0 represent the lowest possible grade, and the biggest possible grade is 1000.

In many compliments, the real physical world is represented more accurately by fuzzy numbers rather than crisp numbers. Assume for example, that you are driving a car on the highway and the speed limit of your car is 55 miles an hour(mph). At 55mph you try to hold your speed limit of car, but your car lacks “cruise control”, so at every moment your car speed varies. If you want to plot a graph speed verses time period and then in rectangular co-ordinates plot the graph, the graph which are shown below your function look like that.

A membership function is a function which is defined in the interval $[0, 1]$. Let μ is any set of space so μ is expressed as $\mu \rightarrow [0,1]$ where each element of this set have membership function between 0 and 1.

3.4Crisp numbers: Crisp numbers are those numbers whose value is exact. When we deals with the membership function its value is either 0 or 1 as the membership function well-defined in $[0,1]$.

3.5Fuzzy Distances: Fuzzy Distance is well-defined as it is the space between two Fuzzy numbers. These distances perform a very important role in the literature.

3.6Crisp Distances: Crisp Distances is well defined as it is the space between two Crisp numbers.

3.7Least square method: Least square process is used to reduce the error and to increase the accuracy of the solution. Least square means that the overall solution minimizes the sum of the squares of the residual made in the results of every single equation.

3.8 Fuzzification: In this process of fuzzification we collect an input of crisp data and convert the data of crisp set into fuzzy set using fuzzy variables.

3.9 Defuzzification: It is a process in which we map a fuzzy set to a crisp set or convert the fuzzy set to crisp set. It is used in fuzzy control system.

4. Methodologies

4.1 Linear Regression

Linear regression equation of Y on X

$$Y = a + b X \quad (4.1)$$

Using the principle of least square, we have to determine the constants **a** and **b** so that:

$$\left. \begin{aligned} \sum_{i=1}^n Y_i &= na + b \sum_{i=1}^n X_i \\ \sum_{i=1}^n X_i Y_i &= a \sum_{i=1}^n X_i + b \sum_{i=1}^n X_i^2 \end{aligned} \right\} \quad (4.2)$$

4.2 Curvilinear regression:

One of the criteria set has been that the variable X and Y are relative linearly. In many cases this assumption may not be valid. For example, in a scatter diagram showing the relationship between X, the number of overtime hours, versus the resulting number of additional produced for a manufacturing firm, it appears by inspection that a curvilinear regression may explain, ore of the variability of Y than does a linear line. For this reason, we shall fit the regression line using the following formula:

4.2.1 Quadratic equation

Quadratic regression equation of Y on X

$$Y = a + b_1 X + b_2 X^2 \quad (4.3)$$

Using the principle of least square , we have to determine the constants a , b_1 and b_2 so that

$$\left. \begin{aligned} \sum_{i=1}^n Y_i &= na + b_1 \sum_{i=1}^n X_i + b_2 \sum_{i=1}^n X_i^2 \\ \sum_{i=1}^n X_i Y_i &= a \sum_{i=1}^n X_i + b_1 \sum_{i=1}^n X_i^2 + b_2 \sum_{i=1}^n X_i^3 \\ \sum_{i=1}^n X_i^2 Y_i &= a \sum_{i=1}^n X_i^2 + b_1 \sum_{i=1}^n X_i^3 + b_2 \sum_{i=1}^n X_i^4 \end{aligned} \right\} \quad (4.4)$$

4.2.2(i) Fitting of a Power Curve: $y = ax^b$

$$y = ax^b \cong \log y = \log a + b \log x \quad (4.5)$$

$$Y = A + bX \quad (4.6)$$

Where $Y = \log y$, $A = \log a$ and $X = \log x$

Using the principle of least square, we have to determine the constants **A** and **b** so that

$$\left. \begin{aligned} \sum_{i=1}^n Y_i &= nA + b \sum_{i=1}^n X_i \\ \sum_{i=1}^n X_i Y_i &= A \sum_{i=1}^n X_i + b \sum_{i=1}^n X_i^2 \end{aligned} \right\} \quad (4.7)$$

4.2.2(ii) Fitting of a Power Curve : $y = ab^x$

$$y = ab^x \cong \log y = \log a + x \log b \quad (4.8)$$

$$Y = A + Bx \quad (4.9)$$

Where $Y = \log y$, $A = \log a$ and $B = \log b$

Using the principle of least square, we have to determine the constants **A** and **B** so that

$$\left. \begin{aligned} \sum_{i=1}^n Y_i &= nA + B \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i Y_i &= A \sum_{i=1}^n x_i + B \sum_{i=1}^n x_i^2 \end{aligned} \right\} \quad (4.10)$$

4.2.3 Fitting of Exponential Curves: $y = ae^{bx}$

$$y = ae^{bx} \cong \log_e y = \log_e a + bx \quad (4.11)$$

$$Y = A + bx \quad (4.12)$$

Where $Y = \log_e y$, and $A = \log_e a$

Using the principle of least square, we have to determine the constants **A** and **B** so that

$$\left. \begin{aligned} \sum_{i=1}^n Y_i &= nA + b \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i Y_i &= A \sum_{i=1}^n x_i + b \sum_{i=1}^n x_i^2 \end{aligned} \right\} \quad (4.13)$$

4.3 Fuzzy regression:

Fuzzy regression is a fuzzy Variation of the regression analysis. This has been studied and applied in lots of areas. We are using the linear regression approach. Its membership function value lies between 0 and 1. It is based upon the observation that people make their decisions on the basis of information not on the basis of data. So to represent that information in the form of data, we are using the fuzzy regression. Fuzzy regression is the set of data in which the values are not exact like in the crisp sets to represent that imprecise data we are using the fuzzy regression. Fuzzy regression is the area of statistics in mathematics in which we deal with the dependence of one variable with another variable. The main objective of the regression analysis is for the approximation of the parameters on the basis of practical data. Demonstrate the fuzzy regression using the data of railway cost/coach with demand value:

$$Y = Cx \quad (4.14)$$

Where $C = \langle c, s \rangle$ the triangular fuzzy members are expressed by these fuzzy parameters. The form of linear programming is algorithm minimize

$$\sum_{i=1}^n s_i \text{ subject to } (1-h) s^T |a_j| - |b_j - a_j^t c| \geq 0 \quad j \in N_m, s_i \geq 0, I \in N_n \quad (4.15)$$

5. DATA AND PROBLEM IDENTIFICATION

The data given below is of The Rail Coach Factory Kapurthala, Punjab for year 2017-2018 for ELEC Section and all classes for LWACCN. The dataset is used in this study is of Linke Hoffmann Bush (LHB) three tier AC coach train. In this study, we will take cost and demand value of all the required things to build a single coach.

The description and specification of all the require things for a coach are given as following, where AC = Air Condition, NAC = Non Air condition, Cu = copper, Cond = conductor:-

Sr.no	Description	Color	Length in SQ MM	CU.COND	Demand Value (X)	Cost/Coach (Y)
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1	Elastic Cable	Black	1.5SQ MM	UP to750V	5083.89	3799
2	Elastic Cable	Blue	1.5 SQ MM	UP to750V	902.72	1215
3	Elastic Cable	Red	1.55 SQ MM	UP to750V	5134.22	3799
4	Elastic Cable	White	1.5 SQ MM	UP to750V	12891.97	2177
5	Elastic Cable	Yellow	1.5 SQ MM	UP to750V	5134.22	2835
6	Elastic Cable	Green/Yellow	1.5 SQ MM	UP to750V	4898.10	1856
7	Elastic Cable	Chocolate	10 SQ MM	UP to750V	3040.00	893
8	Elastic Cable	Blue	2.5 SQ MM	UP to750V	3422.16	416
9	Elastic Cable	Chocolate	2.5 SQ MM	UP to750V	41147.40	934
10	Elastic Cable	Blue	2.5 SQ MM	UP to750V	3422.16	416
11	Elastic Cable	Yellow	2.5 SQ MM	UP to750V	3422.16	614
12	Elastic Cable	Black	25 SQ MM	750V to 1.8/3.0 KV	11816.40	1079
13	Elastic Cable	Black	2.5 SQ MM	UP to750V	19743.66	1973
14	Elastic Cable	Blue	25 SQ MM	750V to 1.8/3.0 KV	11816.40	1079
15	Elastic Cable	Chocolate	25 SQ MM	UP to750V	4864.38	1973
16	Elastic Cable	Red	25 SQ MM	750V to 1.8/3.0 KV	11816.40	1079
17	Elastic Cable	Red	25 SQ MM	UP to750V	19743.66	697
18	Elastic Cable	White	25 SQ MM	UP to750V	4292.10	809
19	Elastic Cable	Blue	25 SQ MM	750V to 1.8/3.0 KV	11816.40	1973
20	Elastic Cable	Yellow	25 SQ MM	UP to750V	19743.66	871
21	Elastic Cable	Blue	25 SQ MM	UP to750V	19743.66	1079
22	Elastic Cable	Blue	4 SQ MM	UP to750V	2152.08	2420
23	Elastic Cable	Green/Yellow	4 SQ MM	UP to750V	722.52	256
24	Elastic Cable	Yellow	4 SQ MM	UP to750V	2152	1463
25	Elastic Cable	Red	4 SQ MM	UP to750V	2152	2178
26	Colored core electron beam outer sheath	Black	4×1.5	600/1000V	94103.30	1977
27	Elastic Cable	Black	50 SQ MM	750V to 1.8/3.0 KV	6262.96	1656
28	Elastic Cable	Black	50 SQ MM	UP to750V	12105.66	1890
29	Elastic Cable	Blue	50 SQ MM	750V to 1.8/3.0 KV	6262.96	1656
30	Elastic Cable	Blue	50 SQ MM	UP to750V	10248.00	1890
31	Elastic Cable	Chocolate	50 SQ MM	UP to750V	1124.56	378
32	Elastic Cable	Green/Yellow	50 MM	UP to750V	2026.53	0
33	Elastic Cable	Red	50 SQ MM	750V to 1.8/3.0 KV	6262.96	1656

34	Elastic Cable	Red	50 SQ MM	UP to750V	12105.66	1890
35	Elastic Cable	White	50 SQ MM	UP to750V	1124.56	378
36	Elastic Cable	Yellow	50 SQ MM	750V to 1.8/3.0 KV	7401.68	1656
37	Elastic Cable	Yellow	50 SQ MM	UP to750V	10248	1890
38	Elastic Cable	Black	6 SQ MM	UP to750V	5057.92	1090
39	Elastic Cable	Green/Yellow	70 SQ	750V to 1.8/3.0 KV	27361.28	521
40	Elastic Cable	Black	95 SQ	750V to 1.8/3.0 KV	80038	5192
	750v. to1.8/3.0 KV					
41	Battery fuse box positive	-	-	-	69500	7776
42	Cable binder	-	-	-	373.80	160
43	Cable binder(3.6×200)	-	3.6×200	-	138	59
44	Cable jacket system	-	-	-	2067	489
45	Cable jacket system	-	-	-	4875	975
46	Cable marking system for LHB EOG type coaches	-	-	---	7139	1428
47	Copper crimping socket for 10SQMM cable	-	10 SQ MM	--	39604.89	33
48	Copper crimping socket for 150SQMM cable	-	150SQ MM	--	3720	645
49	Copper crimping socket for 16SQMM cable	-	16SQ MM	-	43637.86	24
50	Copper crimping socket for 25SQMM cable	-	25SQ MM	-	558.60	32
51	Copper crimping socket for 35SQMM cable	-	33SQ MM	-	261.50	63
52	Copper crimping socket for 50MMSQ cable	-	50 MM SQ	-	947.10	95
53	Copper crimping socket for 50SQMM cable	-	50 SQ MM	-	861	43
54	Copper crimping socket for 6SQMM cable	-	65SQ MM	-	4301.22	7
55	Copper crimping socket for 70SQMM cable	-	75 SQ MM	-	733.44	4,628
56	Emergency lighting system for EOG type LHB coaches	-	-	-	71,125	13216
57	Ext Sup socket 125 AMP,415V, IP67,5pole(3P+N+E) With piolet contacts ,RCF ANNX A for external supply socket	-	-	-	17,950	3802

58	F.R.L.T TAPE	Low	Tension	Cot	INSUL	Green	0.3 MM roll of 10M length	-	385.86	79
59	F.R.L.T TAPE	Low	Tension	Cot	INSUL	Red	0.3 MM roll of 10M length	-	257.24	52
60	F.R.L.T TAPE	Low	Tension	Cot	INSUL	Black	0.3 MM roll of 10M length	-	392.94	79
61	F.R.L.T TAPE	Low	Tension	Cot	INSUL	Blue	10M	-	392.94	79
62	F.R.L.T TAPE	Low	Tension	Cot	INSUL	Yellow	0.3MM	-	392.94	79
63	FIXING CLIP 72MM DIA					-	72MM	-	1340	2572

Table5.1. Data of railway demand value and cost/coach

$$\sum X=777829.79$$

$$\sum Y=98018$$

$$\sum X^2=34169320871$$

$$\sum Y^2=419200636$$

$$\sum XY=2643420642$$

$$\bar{X}=12427.20$$

$$\bar{Y}=1555.8$$

6. Result Analysis

Linear Regression equation of Y on X

Using the equations 4.1 and 4.2 we get $Y = -3949.40 + 0.3443X$ (6.1)

$$y=a+bx$$

$$y= 3949.40+0.3443 X$$

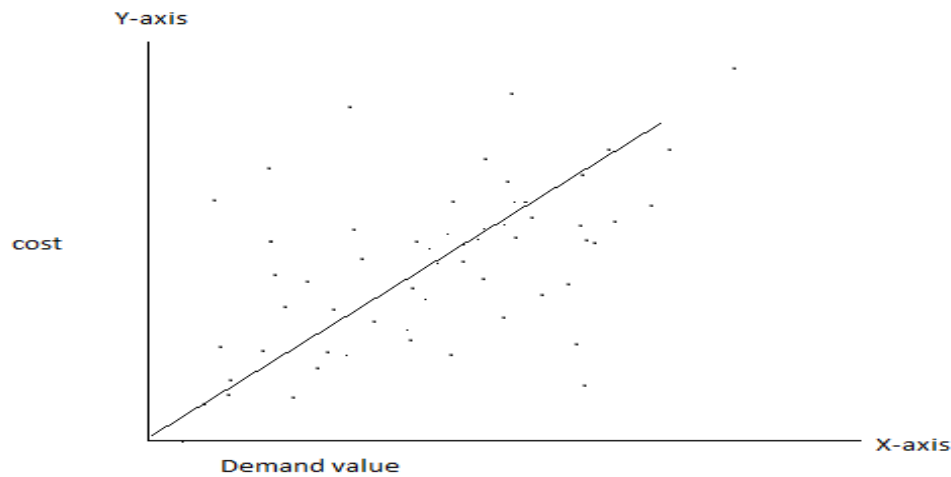
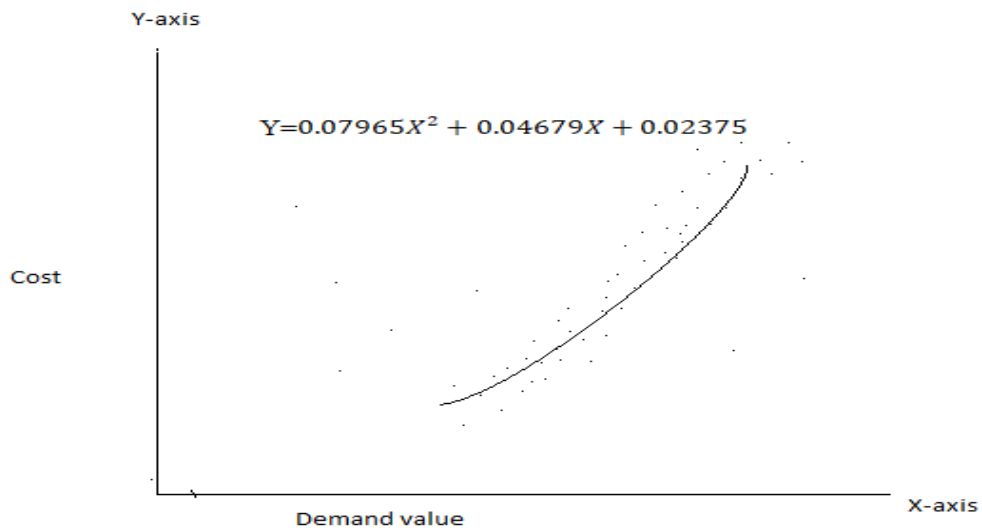


Figure 6.1 Linear Regression of Demand value versus cost/coach

Quadratic regression equation:

Using the equations 4.3 and 4.4, we get $Y=0.07965X^2 + 0.04679X + 0.02375$ (6.2)



Figure

6.2 Quadratic graph between demand value and cost/coach

Fitting of a Power Curve:

Using the equations 4.5, 4.6 and 4.7, we get $Y = 0.04964X^{0.02791}$ (6.3)

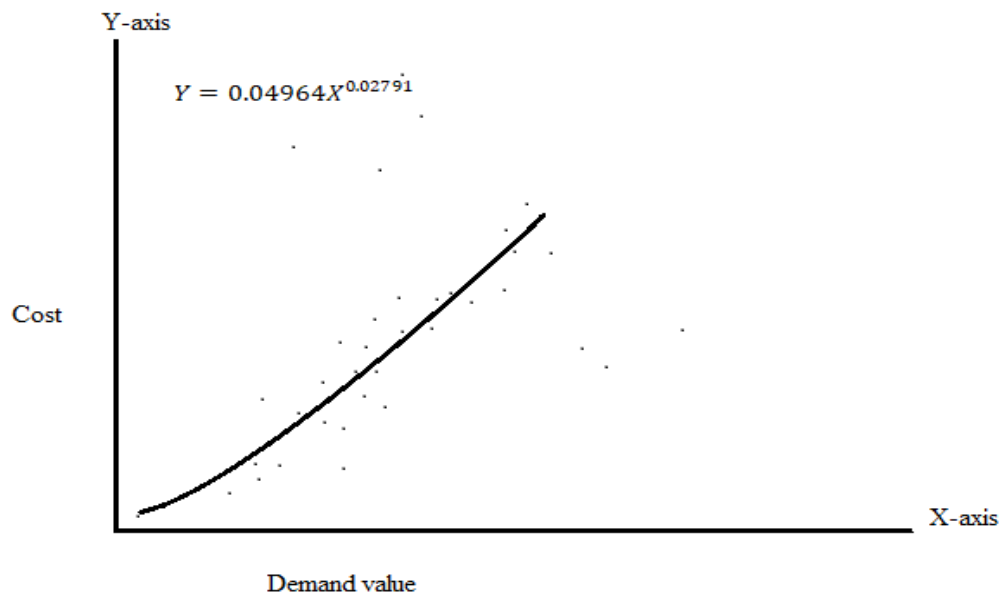


Figure 6.3 Power curve graph between Demand value and cost/coach

Fitting of Exponential Curves:

Using the equations 4.8, 4.9 and 4.10, we get $Y=0.0698 \times 0.0277^X$ (6.4)

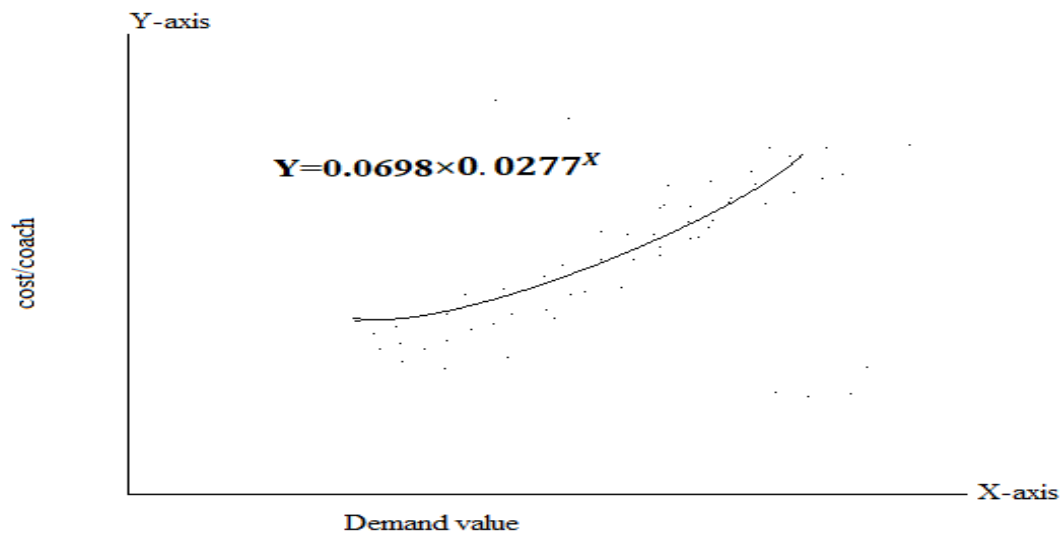


Figure 6.4 Exponential graph between demand value and cost/coach

Using the equations 4.11, 4.12 and 4.13, we get $Y=0.08463e^{0.04973X}$

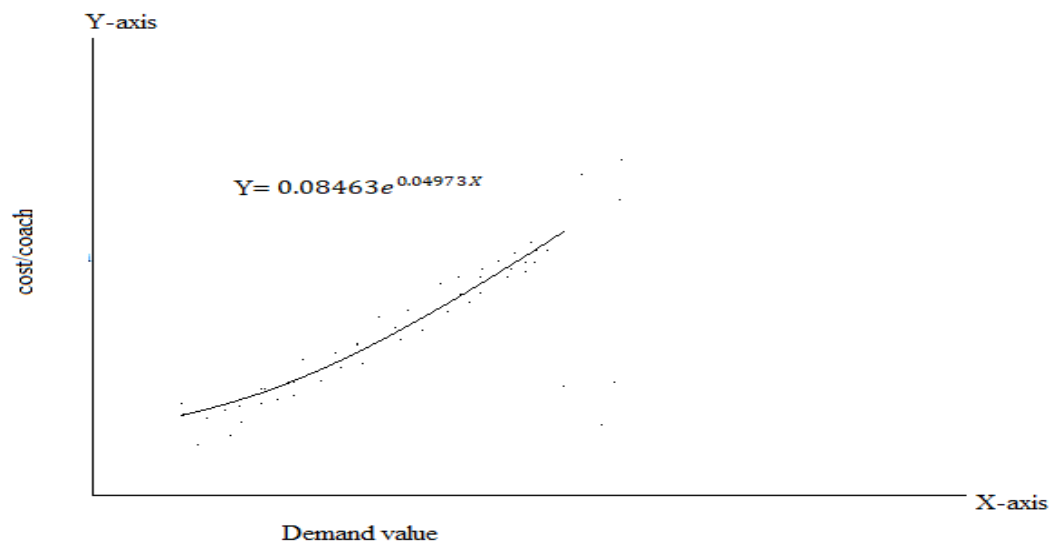


Figure 6.5 Exponential graph between demand value and cost/coach

Demonstrate the fuzzy regression using the data of railway cost/coach with demand value:

Using the demand value and cost/coach equations we state the values of c, s, C:

When $h=1/2c=3.1549$ $s=6.3098$ $C=(c, s)$ $C=(3.1549, 6.3098)$

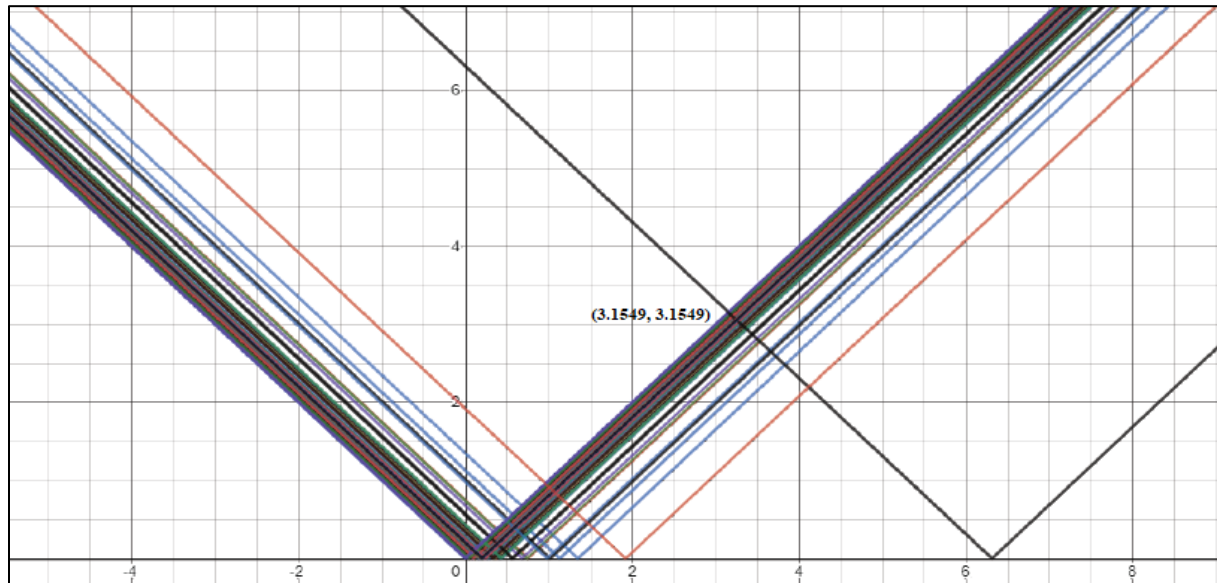


Figure 6.6 Fuzzy Regression graph of Demand value versus cost/coach

Demand Value (X)	Cost/Coach (Y)	c	s	Base	Prob.Inc
5083.89	3799	3.155091	6.309991	16040.14	23971.66
902.72	1215	3.155091	6.309991	2848.164	7666.639
5134.22	3799	3.155091	6.309991	16198.93	23971.66
12891.97	2177	3.155091	6.309991	40675.34	13736.85
5134.22	2835	3.155091	6.309991	16198.93	17888.82
4898.1	1856	3.155091	6.309991	15453.95	11711.34
3040	893	3.155091	6.309991	9591.477	5634.822
3422.16	416	3.155091	6.309991	10797.23	2624.956
41147.4	934	3.155091	6.309991	129823.8	5893.532
3422.16	416	3.155091	6.309991	10797.23	2624.956
3422.16	614	3.155091	6.309991	10797.23	3874.334
11816.4	1079	3.155091	6.309991	37281.82	6808.48
19743.66	1973	3.155091	6.309991	62293.05	12449.61
11816.4	1079	3.155091	6.309991	37281.82	6808.48
4864.38	1973	3.155091	6.309991	15347.56	12449.61
11816.4	1079	3.155091	6.309991	37281.82	6808.48
19743.66	697	3.155091	6.309991	62293.05	4398.064
4292.1	809	3.155091	6.309991	13541.97	5104.783
11816.4	1973	3.155091	6.309991	37281.82	12449.61
19743.66	871	3.155091	6.309991	62293.05	5496.002
19743.66	1079	3.155091	6.309991	62293.05	6808.48
2152.08	2420	3.155091	6.309991	6790.009	15270.18
722.52	256	3.155091	6.309991	2279.617	1615.358
2152	1463	3.155091	6.309991	6789.756	9231.517
2152	2178	3.155091	6.309991	6789.756	13743.16
94103.3	1977	3.155091	6.309991	296904.5	12474.85
6262.96	1656	3.155091	6.309991	19760.21	10449.35

12105.66	1890	3.155091	6.309991	38194.46	11925.88
6262.96	1656	3.155091	6.309991	19760.21	10449.35
10248	1890	3.155091	6.309991	32333.38	11925.88
1124.56	378	3.155091	6.309991	3548.089	2385.177
2026.53	0	3.155091	6.309991	6393.887	0
6262.96	1656	3.155091	6.309991	19760.21	10449.35
12105.66	1890	3.155091	6.309991	38194.46	11925.88
1124.56	378	3.155091	6.309991	3548.089	2385.177
7401.68	1656	3.155091	6.309991	23352.98	10449.35
10248	1890	3.155091	6.309991	32333.38	11925.88
5057.92	1090	3.155091	6.309991	15958.2	6877.89
27,361.28	521	3.155091	6.309991	86327.34	3287.505
80038	5192	3.155091	6.309991	252527.2	32761.47
69500	7776	3.155091	6.309991	219278.8	49066.49
373.8	160	3.155091	6.309991	1179.373	1009.599
138	59	3.155091	6.309991	435.4026	372.2895
2067	489	3.155091	6.309991	6521.574	3085.586
4875	975	3.155091	6.309991	15381.07	6152.241
7139	1428	3.155091	6.309991	22524.2	9010.667
39604.89	33	3.155091	6.309991	124957	208.2297
3720	645	3.155091	6.309991	11736.94	4069.944
43637.86	24	3.155091	6.309991	137681.4	151.4398
558.6	32	3.155091	6.309991	1762.434	201.9197
261.5	63	3.155091	6.309991	825.0564	397.5294
97.1	95	3.155091	6.309991	306.3594	599.4491
861	43	3.155091	6.309991	2716.534	271.3296
4301.22	7	3.155091	6.309991	13570.74	44.16994
733.44	4,628	3.155091	6.309991	2314.07	29202.64
71,125	13216	2.969086	6.309991	211176.3	83392.84
17,950	3802	2.953851	6.309991	53021.63	23990.59
385.86	79	2.953851	6.309991	1139.773	498.4893
257.24	52	2.953851	6.309991	759.8488	328.1195
392.94	79	2.953851	6.309991	1160.686	498.4893
392.94	79	2.953851	6.309991	1160.686	498.4893
392.94	79	2.953851	6.309991	1160.686	498.4893
1340	2572	1.235497	6.309991	1655.566	16229.3

Table 6.1: Shows the fuzzy regression of demand and cost/coach

Conclusion

In this paper, we investigated linear regression and fuzzy linear regression with fuzzy coefficients on the demand, cost/coach for the coach wise material schedule for the year 2017-18 for ELEC sections and all classes for LWACCN. Firstly, we represent the data using different methods like power curve, exponential curve and so many other curves which is very helpful to understand analysis of the data and then fuzzy regression based on fuzzy coefficients was evaluated. The fluctuated parameters (slope) were evaluated with respect to basic values of slope

of all data set, which shows that more degree of reliability as compared to different methods of regression.

Reference

- [1] A. Bárdossy, “Note on fuzzy regression,” *Fuzzy Sets and Systems*, 1990.
- [2] H. Tanaka, J. Watada, and K. Asai, “Linear Regression Analysis by Possibilistic Models,” in *Optimization Models Using Fuzzy Sets and Possibility Theory*, 1987.
- [3] B. Heshmaty and A. Kandel, “Fuzzy linear regression and its applications to forecasting in uncertain environment,” *Fuzzy Sets and Systems*, 1985.
- [4] D. A. Savic and W. Pedrycz, “Evaluation of fuzzy linear regression models,” *Fuzzy Sets and Systems*, 1991.
- [5] D. A. Freedman, *Statistical models: Theory and practice*. 2009.
- [6] D. Freedman and D. Freedman, “The Regression Line,” in *Statistical Models: Theory and Practice*, 2012.
- [7] T. Speed, “Statistical Models: Theory and Practice, Revised Edition by David A. Freedman,” *International Statistical Review*, 2010.
- [8] B. Kim and R. R. Bishu, “Evaluation of fuzzy linear regression models by comparing membership functions,” *Fuzzy Sets and Systems*, 1998.
- [9] Y. R. Fan, G. H. Huang, and A. L. Yang, “Generalized fuzzy linear programming for decision making under uncertainty: Feasibility of fuzzy solutions and solving approach,” *Information Sciences*, vol. 241, pp. 12–27, 2013.
- [10] P. D’Urso, “Linear regression analysis for fuzzy/crisp input and fuzzy/crisp output data,” *Computational Statistics and Data Analysis*, vol. 42, no. 1–2, pp. 47–72, 2003.
- [11] H. L. Li and J. R. Yu, “A piecewise regression analysis with automatic change-point

- detection,” *Intelligent Data Analysis*, 1999.
- [12] J. R. Yu, G. H. Tzeng, and H. L. Li, “General fuzzy piecewise regression analysis with automatic change-point detection,” *Fuzzy Sets and Systems*, 2001.
- [13] J. R. Yu, G. H. Tzeng, and H. L. Li, “Interval piecewise regression model with automatic change-point detection by quadratic programming,” *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 2005.
- [14] J. R. Yu and C. W. Lee, “Piecewise regression for fuzzy input-output data with automatic change-point detection by quadratic programming,” *Applied Soft Computing Journal*, 2010.
- [15] M. S. Yang and T. S. Lin, “Fuzzy least-squares linear regression analysis for fuzzy input-output data,” *Fuzzy Sets and Systems*, 2002.
- [16] A. F. Roldán López De Hierro, J. Martínez-Moreno, C. Aguilar-Peña, and C. Roldán López de Hierro, “Estimation of a Fuzzy Regression Model Using Fuzzy Distances,” *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 2, pp. 344–359, 2016.
- [17] A. Azadeh, M. Saberi, and O. Seraj, “An integrated fuzzy regression algorithm for energy consumption estimation with non-stationary data: A case study of Iran,” *Energy*, 2010.
- [18] Z. Xu, K. Gao, T. M. Khoshgoftaar, and N. Seliya, “System regression test planning with a fuzzy expert system,” *Information Sciences*, 2014.