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Some Results on 2-odd Labeling of Graphs

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Abstract:

A graph G is said to be a 2-odd graph if the vertices of G can be labelled with integers (necessarily distinct) such that for any two vertices which are adjacent, then the modulus difference of their labels is either an odd integer or 2. In this paper, we investigate 2-odd labeling of some graphs.

Keywords*: Distance Graphs, 2-odd Graphs*

1. Introduction

All the graphs discussed in this article are connected, simple, non-trivial, finite, and undirected. By $G(V,E)$, or simply G, we mean a graph G whose vertex set is V and edge set is E. According to J.D. Laison et al. [2] a graph G is 2-odd if there exists a $1-1$ labeling $h: V(G) \rightarrow Z$ ("the set of all integers") such that for any two vertices u and v which are adjacent, the integer $|h(u)-h(v)|$ is either 2 or odd. They also defined that $h(uv) = |h(u)-h(v)|$ and called h a 2-odd labeling of G. So G is a 2-odd graph iff there exists a 2-odd labeling of G. One can notice that in a 2-odd labeling, the vertex labels of G are distinct, but edge labels are not so. Moreover, by this definition, $h(uv)$ may still be either 2 or odd if uv is not an edge of G. The stronger version of 2-odd labeling is known as a prime distance labeling of graphs. For a detailed study on prime distance graphs one can refer to [4, 5, 6, 7, and 8]. In this present paper, we investigate 2-odd labeling of certain graphs.

Note 1:
A 2-odd labeling of a graph G is not unique.

Example 1:
A path P_n can be labelled with $2, 4, \ldots, 2n$ or $3, 6, \ldots, 3n$.

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2. Main Results

In this section, we establish 2-odd labeling of some graphs such as wheel graph, fan graph, generalized butterfly graph, the line graph of sunlet graph, and shell graph.

Lemma 1.

If a graph G has a subgraph that does not admit a 2-odd labeling then G cannot have a 2-odd labeling.

One can note that the Lemma 1 clearly holds since if G has a 2-odd labeling then this will yield a 2-odd labeling for any subgraph of G.

Definition 1: [5]

The wheel graph $W_n = C_{n-1} \wedge K_1$ is a graph with n vertices ($n \geq 4$), formed by joining the central vertex K_1 to all the vertices of C_{n-1}.

Theorem 1:

The wheel graph W_n admits a 2-odd labeling for all $n \geq 4$.

Proof.

Let W_n be the given wheel graph on $n \geq 4$ vertices. It is clear to see that there are $n-1$ vertices on the rim, namely $v_1, v_2, \ldots, v_n$ and the central vertex, say v_0. Now define an injective function $f: V(W_n) \to Z$ as follows: without loss of generality, let $f(v_0) = 1$, $f(v_1) = 3$, $f(v_2) = 4$, and $f(v_i) = f(v_i) + 2$ for $3 \leq i \leq n$. An easy check shows that f is the required 2-odd labeling of W_n as $|f(v_0) - f(v_i)|$ are all odd numbers for $3 \leq i \leq n$, $|f(v_0) - f(v_1)| = 2$, $|f(v_0) - f(v_2)| = 3$, $|f(v_i) - f(v_{i+1})| = 2$ for $2 \leq i \leq n-1$, and $|f(v_n) - f(v_1)|$ is an odd integer.

Definition 2: [5]

A fan graph $F_{(1,n)}$ is defined as the graph $K_1 \wedge P_n$, where K_1 is the empty graph on one vertex and P_n is a path on n vertices.

Theorem 2:

The fan graph $F_{(1,n)}$ admits a 2-odd labeling for any $n \geq 1$.

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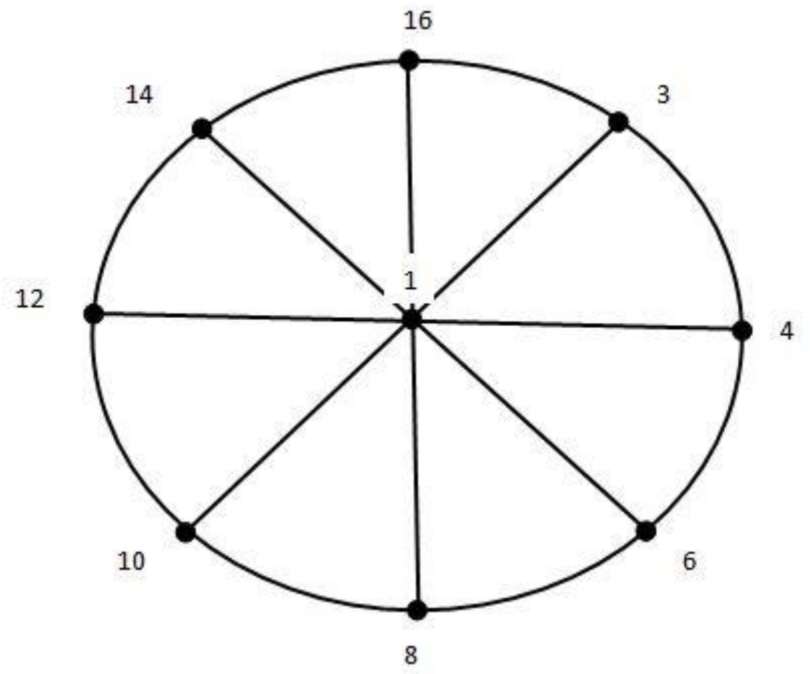

Figure 1. A 2-odd labeling of W_9

Proof.

Let $F_{(1,n)}$ be the given fan graph on $n \geq 1$ vertices. We label the vertex K_1 as v_0 and vertices of a path P_n as $v_1, v_2, \ldots, v_n$. One can note that $|F_{(1,n)}| = n + 1$. Now define an injective labeling $h: V(F_{(1,n)}) \to Z$ as follows: without loss of generality, let $h(v_0) = 1$ and $h(v_i) = 2i$ for $1 \leq i \leq n$. An easy check shows that h is the desired 2-odd labeling of $F_{(1,n)}$.

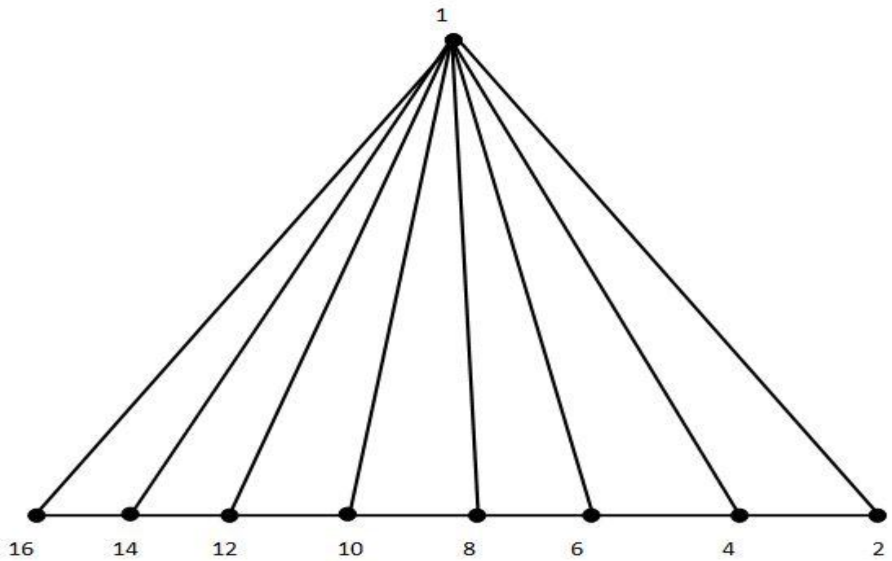

Figure 2. A 2-odd labeling of $F_{(1,8)}$

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Definition 3: [1]

The generalized butterfly graph, denoted BF_n, is a graph formed by adjoining vertices to every wing with the assumption that "sum of inserting vertices to every wing are same."

Theorem 3:

The generalized butterfly graph BF_n is a 2-odd graph for $n \geq 3$.

Proof.

Let $BF_n, n \geq 3$ be the given generalized butterfly graph. One can notice that $|V(BF_n)| = 2n + 1$ and $|E(BF_n)| = 4n - 2$. Let $V(BF_n)$ be $\{v_i : i = 1, 2, \ldots, 2n\}$ and $E(BF_n)$ be $\{(v_i, v_{i+1}) : i = 1, 2, \ldots, n - 1, n + 1, \ldots, 2n - 1\} \cup \{(v_0, v_i) : i = 1, 2, \ldots, 2n\}$. We label the apex vertex as v_0, the vertices on right wing as $v_1\ v_2, \ldots, v_n$, and the vertices on left wing as $v_{n+1}\ v_{n+2}, \ldots, v_{2n}$. Define a one-one labeling $h: V(BF_n) \to Z$ as follows: without loss of generality, let $h(v_0) = 2m + 1$, for any $m \in N$ and $h(v_i) = 2i$ for $1 \leq i \leq 2n$. One can easily see that h is the required 2-odd labeling of BF_n.

Definition 4: [9]

The sunlet graph S_n is a graph on $2n$ vertices obtained by attaching n −pendant edges to the cycle C_n.

Definition 5: [9]

The line graph of a graph G, denoted by $L(G)$, is a graph whose vertices are the edges of G, and if $uv \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G.

Theorem 4:

The line graph of a sunlet graph $L(S_n)$ is a 2-odd graph for $n \geq 3$.

Proof.

Let $S_n, n \geq 3$ be the given sunlet graph on $2n$ vertices. Obtain the line graph of a sunlet graph $L(S_n)$ whose vertex set is defined as $V\big(L(S_n)\big) = V_1 \cup V_2$ where $V_1 = \{v_1, v_2, \ldots, v_n\}$ and $V_2 = \{u_1, u_2, \ldots, u_n\}$. Now define a one-to-one labeling $h: V(L(S_n)) \to Z$ as follows: without loss of generality, let $h(v_i) = 2i$ for $1 \leq i \leq n - 1$, $h(v_n) = -1$, $h(u_i) = 2(i - 1) + 1$ for $1 \leq i \leq n - 1$, and $h(u_n) = h(u_{n-1}) + 3$. One can see that h is the required 2-odd labeling of $L(S_n)$.

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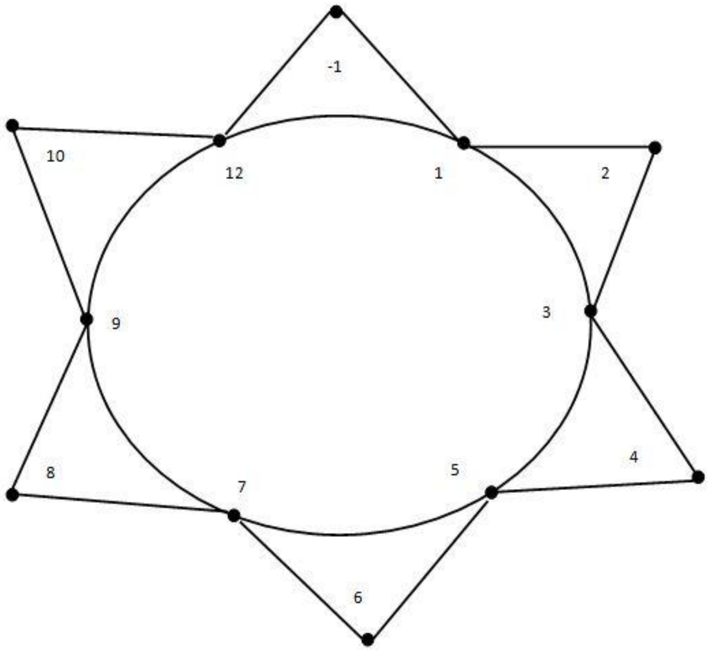

Figure 3. A 2-odd labeling of $L(S_6)$

Definition 6: [3]

A shell graph $C(n, n-3)$ is defined as a cycle C_n with $(n-3)$ chords sharing a common end vertex called the apex.

Theorem 5:

The shell graph $C(n, n-3)$ permits a 2-odd labeling for all $n \geq 4$.

Proof.

Let $C(n, n-3)$ be the given shell graph with $n \geq 4$. We label the vertices of $C(n, n-3)$ as $v_1, v_2, \ldots, v_n$ and let v_1 be the apex vertex. Define an injective labeling $h: V(C(n, n-3)) \rightarrow Z$ as follows: without loss of generality, let $h(v_1) = 1$ and $h(v_i) = 2h(v_{i-1})$ for $2 \leq i \leq n$. Clearly h is the required 2-odd labeling of $C(n, n-3)$.

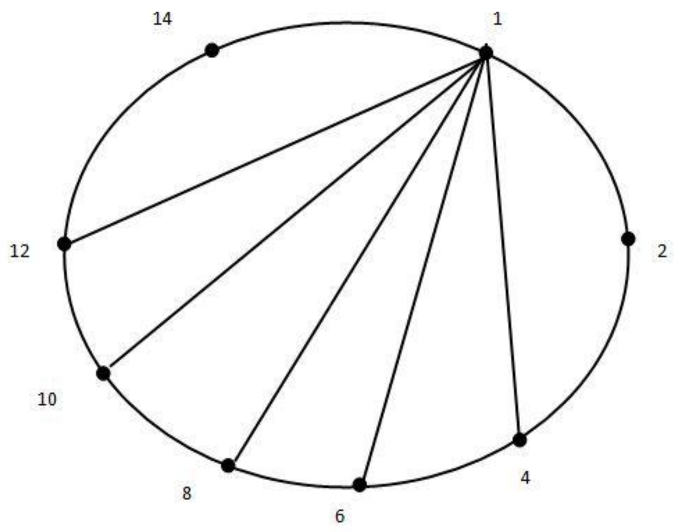

Figure 4. A 2-odd labeling of $C(8,5)$

3. Conclusion

The 2-odd labeling of various classes of graphs such as wheel graph, fan graph, generalized butterfly graph, line graph of sunlet graph, and shell graph are established. Investigating 2-odd labeling of other classes of graphs is still open and this is for future work. One can also explore the exclusive applications of 2-odd labeling in real life problems.

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