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On a Kählerian manifold

Pankaj Pandey and E.S. Lone

Department of Mathematics, School of Chemical Engineering and Physical Sciences,

Lovely Professional University, Phagwara-144411, Punjab

Email: pankaj.anvarat@gmail.com

sabir.11900873@lpu.in

Abstract: In this paper, we have studied different type of curvature tensors and some properties of curvature tensors have been discussed.

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1. Introduction

In 1975, a linear connection is introduced by S. Golab [8] called quarter-symmetric connection. Golab [8] defined that condition $T(X,Y) = \omega(Y)\varphi X\ \omega(X)\varphi Y$, for tensor field φ of type (1,1) and X,Y are arbitrary vector fields. A linear connection is a non-metric connection if covariant derivative of the Riemannian metric g does not vanish i.e. $\nabla g \neq 0$.

The semi-symmetric connection under non-metric condition has been studied in [6] (2008) in a Kähler manifold. Recently, some results have been obtained in weakly symmetric manifold in [5] (2017). Further they studied semi-symmetric non-metric connection [10] (2017) and quarter-symmetric non-metric connection [9] (2018) on lift of Kähler manifolds [10]. Nijenhuis tensor becomes zero in a Kähler manifold [6]. In the same paper, they [6] obtained some results related to contra-variant vector field in Kähler manifold with semi-symmetric connection. Flatness of Kähler manifold under weak symmetric condition has been discussed in [1] (2014). In 2015 a new type of results has been studied [2] in an almost Hermitian manifold. The same authors [3] further studied a special type of Kähler manifold under weak symmetric condition. The study of

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conformal connection has been extended by Chaturvedi and Pandey [4] in an almost Hermitian manifold in 2016.

Let M be differentiable manifold of dimension 2k, for non-negative integer k. If the conditions

$$F^2\,(X) + X = 0,\; g(FX, FY) = g(X, Y),\, (\nabla_X F)Y = 0, \tag{1.1}$$

hold then M becomes Kähler manifold.

In this paper, we have considered a connection ∇^*

$$\nabla^*_X Y = \nabla_X Y + \omega(Y)FX \tag{1.2}$$

satisfying

$$(\nabla^*_X g)(Y, Z) = \alpha[\omega(Y)g(FX, Z) + \omega(Z)g(FX, Y) \tag{1.3}$$

$$T^*(X, Y) = \omega(Y)FX - \omega(X)FY \tag{1.4}$$

respectively for ω 1-form defined by $\omega(X) = g(X, \rho)$, where ρ is an associated vector field.

2. Riemannian and Ricci Curvature Tensors

Let M denotes a manifold of dimension 2k, for non negative integer, then Riemannian tensor is defined by

$$R(A, B, P) = \nabla_A \nabla_B P - \nabla_B \nabla_A P - \nabla_{[A,B]} Z \tag{2.1}$$

and the Ricci tensor is contraction of R.

Now, equation (2.1) becomes for ∇^*

$$R^*(X, Y, Z) = \nabla^*_X \nabla^*_Y Z - \nabla^*_Y \nabla^*_X Z - \nabla^*_{[X,Y]} Z \tag{2.2}$$

Using (1.2) in (2.2), we get

$$R^*(A, B, P) = R(A, B, P) + [(\nabla_A \omega)(P)FB - (\nabla_B \omega)(P)FA] + [(\nabla_A F)B - (\nabla_B F)A + \omega(FB)FA - \omega(FA)FB]\omega(P). \tag{2.3}$$

If the associated vector field is unit parallel then $\nabla_X \rho = 0$, which imply

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$$(\nabla_P \omega)(Z) = 0. \quad (2.4)$$

Now, using (1.1) and (2.4) in (2.3), we get

$$R^*(A, B, P) = R(A, B, P) + [\omega(FB)FA - \omega(FA)FB]\omega(P). \quad (2.5)$$

Hence, result can be stated as

$\boldsymbol{Theorem}$ **2.1** *For M being a Kähler manifold and ∇^* defined by (1.2) then for a associated parallel unit vector field ρ the Riemmanian curvature tensor is* given by (2.6).

Now contracting (2.5), we get

$$S^*(B, P) = S(B, P) + \omega(B)\omega(P) \quad (2.6)$$

Hence, we obtained the result

$\boldsymbol{Theorem}$ **2.2** *For M being a Kähler manifold and ∇^* defined by (1.2) then for a associated parallel unit vector field ρ the Ricci curvature tensor is* given by (2.6).

Further, Contracting (2.6), we get

$$r^* = r \quad (2.7)$$

Therefore, we find result

$\boldsymbol{Theorem}$ **2.3** *For M being a Kähler manifold and ∇^* defined by (1.2) then for a associated parallel unit vector field ρ (2.7) holds.*

3. Properties of Riemannian and Ricci curvature tensors

The Riemannian curvature tensor R with respect to Riemannian connection ∇ satisfies the following properties:

(i) $R(FA, FP)Z = R(A, P)Z$

(ii) $R(A, S) = -R(S, A)$

Now, equation (2.6) implies

$$R^*(FA, FB, P) = R(A, B, P) + [\omega(B)A - \omega(A)B]\omega(P). \quad (3.1)$$

Subtracting (2.6) from (3.1), we have

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$$R^*(A,B,P) - R(A,B,P) = [\omega(B)A - \omega(A)B - \omega(FB)FA + \omega(FA)FB]\omega(P) \tag{3.2}$$

If we denote

$$D(A,B) = \omega(FB)FA - \omega(B)A \tag{3.3}$$

then we get

$$D(FP,FQ) + D(P,Q) = \omega(B)A - \omega(A)B - \omega(FB)FA + \omega(FA)FB \tag{3.4}$$

Hence, we can state

***Theorem* 3.1** *For M being a Kähler manifold and ∇^* defined by (1.2) then for a associated parallel unit vector field ρ then Riemannian curvature tensor $R^*$is* Kählerian if $D(A,B)$is skew Hermitian that is $D(FA,FB) = -D(A,B)$.

Further, we can easily obtained

$$R^*(A,B) = -R^*(B,A)$$

Hence, we have

***Theorem* 3.2** *For M being a Kähler manifold and ∇^* defined by (1.2) then for a associated parallel unit vector field ρ then R^* is* skew symmetric in first two slots.

References:

[1] B.B. Chaturvedi and P. Pandey, On the Flatness of Weakly Symmetric Kähler Manifolds, Journal of International Academy of Physical Sciences, 18 (4), (2014), 279-284.

[2] B.B. Chaturvedi and P. Pandey, Study of a new type of metric connection in an almost Hermitian manifold, Facta Universitatis (NIS), Series: Mathematics and Informatics, 30(2) (2015), 217-224.

[3] B.B. Chaturvedi and P. Pandey, Study on special type of a weakly symmetric Kähler manifold, Differential Geometry Dynamical system 17 (2015), 32-37.

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[4] B.B. Chaturvedi and P. Pandey, Conformal transformation in an almost Hermitian manifold, Novi Sad Journal of Mathematics, 46 (1), (2016), 217-226.

[5] B.B. Chaturvedi and P. Pandey, Weak symmetric manifolds equipped with semi-symmetric metric connection, Palestine Journal of Mathematics, 6 (2), (2017), 507-514.

[6] B.B. Chaturvedi and P.N. Pandey, A semi-symmetric non-metric connection on a Kähler manifold, Differential Geometry Dynamical system, 10 (2008), 86-90.

[7] U.C. De and A.A. Shaikh, Complex Manifolds and Contact Manifolds, Narosha Publishing House, 2009.

[8] S. Golab, On semi-symmetric and quarter-symmetric linear connections, Tensor, N. S., 29 (1975), 249-254.

[9] P. Pandey and B.B. Chaturvedi, On a Kähler manifold equipped with lift of quarter symmetric non-metric connection, Facta Universitatis (NIS), Series: Mathematics and Informatics, 33(4), (2018), 539–546.

[10] P. Pandey and B.B. Chaturvedi, Study of Kähler manifolds endowed with lift of semi-symmetric non-metric connection, Acta Mathematica Academiae Paedagogicae Nyregyhaziensis, 33 (2017), 61-66.